LESSON 9

REARRANGING EQUATIONS OF LINES

LEARNING TARGETS

- *I can rearrange the equation of a line into different forms.*
- *I can relate key features of lines to different forms of linear equations.*

ALIGNMENT

Addressing

- **A1.PAFR.2.1** Transform linear, quadratic, exponential, and linear absolute value functions to equivalent forms to identify slope and y-intercept for linear, vertex, and roots (if any) for quadratic and linear absolute value and y-intercept for exponential.

LESSON PREPARATION

Required Materials

- Cool Down

LESSON NARRATIVE

Students extend their understanding of rearranging formulas to rewrite equations of lines in different but equivalent forms. Throughout the lesson, they rearrange equations from standard form, point-slope form, and slope-intercept form and discuss how they represent the same relationship. This is students' first formal introduction to point-slope form and standard form of linear equations. Students have worked with slope-intercept form throughout 8th grade and Geometry with Statistics and will have many opportunities to reason quantitatively and abstractly (MPS.AJ.1), using contextually appropriate mathematical language and models (MPS.RC.1).

WARM-UP | SOLVING EQUATIONS *5 minutes*

Instructional Routines

- Think-Pair-Share

This Warm-Up revisits students' previous knowledge of solving one-variable linear equations. Encourage students to list all their steps for solving the equation. Seeing these steps will be important later in this lesson as they notice patterns between the steps used to solve one-variable equations and the steps used to rearrange an equation of a line (MPS.SP.1).

LAUNCH

Arrange students in groups of 2. Give students 2–3 minutes (min.) of quiet work time to complete the Warm-Up, followed by 1 min. to share their responses with their partners. Make note of students who solve the equations differently, and highlight their work in a whole-class discussion after the partner discussion.

STUDENT ACTIVITY

Solve each equation by isolating the variable. Show your work.

a. $14 = 3x - 28$
b. $x + 19 = 6(x - 11)$
c. $27 + 9y = -12$

Possible Responses

Sample responses:

a. $14 = 3x - 28$
$42 = 3x$
$14 = x$

b. $x + 19 = 6\,(x - 11)$
$x + 19 = 6x - 66$
$-5x + 19 = -66$
$-5x = -85$
$x = 17$

c. $27 + 9y = -12$
$9y = -39$
$y = -\frac{13}{3}$

Activity Synthesis

Invite previously identified students to share their steps and solutions to the equations. Discuss with the whole class how each step in isolating the variable creates a new equivalent equation in the process.

GUIDED ACTIVITY | DIFFERENT FORMS OF LINEAR EQUATIONS *15–18 minutes*

Instructional Routines

- Think-Pair-Share
- MLR3: Clarify, Critique, Correct

This activity introduces students to additional forms of two-variable linear equations—point-slope form and standard form. It is a Guided Activity and provides the opportunity to use direct instruction to introduce each of the forms. Note this does not mean it is not intended to be collaborative with opportunities for students to discuss and make sense of the representations. There are opportunities embedded throughout the activity to introduce new ideas and then pause to give students time to discuss and make connections using their prior knowledge of lines and linear relationships (MPS.PS.1). These discussion opportunities also provide the chance to make informal observations about students' understanding of linear relationships before moving forward in the lesson.

LAUNCH

Arrange students in groups of 2. Tell students, "Although you are familiar with slope-intercept form from prior courses, there are additional forms of two-variable linear equations that we will explore in this component. These forms are generally referred to as point-slope form and standard form, which we'll discuss during the activity." Display the 3 equations and graphs of the line, and give students 1 min. to discuss problem 1 with their partners. Then, use the introduction and slope-intercept form to review what students have learned in prior courses before giving them a chance to discuss and complete problem 2 with their partners.

Provide students with 1 min. to complete problem 3, followed by a partner discussion to share things they noticed. Guide students through problems 4a–4c using a think-aloud technique to share your thought process and questioning with the class. Be sure to connect the steps of rearranging the equation to the steps of solving a one-variable linear equation. Allow students 1–2 min. of quiet work time to complete 4d and 4e. Bring the class together by guiding students through problem 4d and having 2–3 students share what they notice in problem 4e. Review standard form, and guide students through problem 5. Consider starting problem 5 and pausing to allow students to finish rewriting the equation before you guide them through the rest of the problem. Allow students 1 min. of quiet work time to complete problem 6, followed by a partner discussion to share responses.

Support for English Language Learners

Conversing: MLR 3 Clarify, Critique, Correct. Present incorrect work for rearranging the equation in problem 4d. Ask students to identify the error(s) and correct the work. This will help students to build a connection when rearranging an equation of a line.

Design Principle(s): Optimize output (for generalization); Maximize meta-awareness

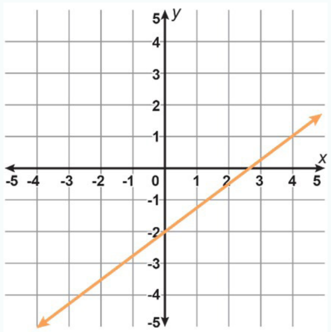
Support for Students with Disabilities

Receptive Language: Processing Time. Read all statements aloud. Students who both listen to and read the information will benefit from extra processing time.

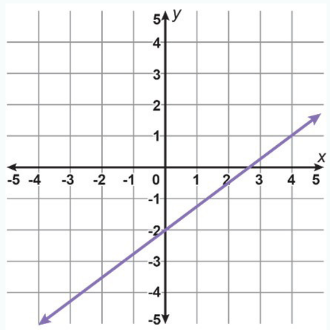
STUDENT ACTIVITY

Three linear equations and their graphs are shown.

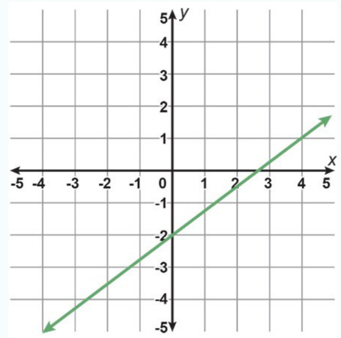
$y = \frac{3}{4}x - 2$



$(y - 1) = \frac{3}{4}(x - 4)$

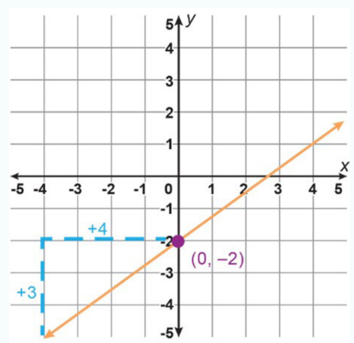


$3x - 4y = 8$



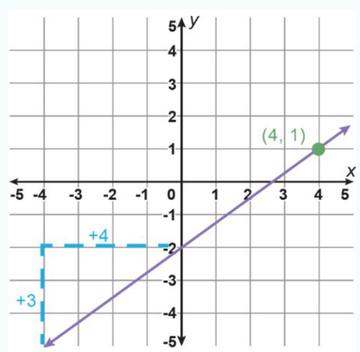
1. Discuss with your partner what you notice about the graphs and equations
The equations of lines can be written in different but equivalent forms. Each form of the equation relates to different features of the graph, including the slope, y-intercept, and other points on the line.

One form for the equation of a line is *slope-intercept form*.

Slope-Intercept Form	$y = \frac{3}{4}x - 2$
$y = mx + b$ where m is the slope and b is the y-intercept	

2. Discuss with your partner why this form is called slope intercept form. Summarize your discussion.

A second form for the equation of a line is called *point-slope form.*

Point-Slope Form	$(y - 1) = \frac{3}{4}(x - 4)$
$(y - y_1) = m(x - x_1)$ where m is the slope and (x_1, y_1) is any point on the line	

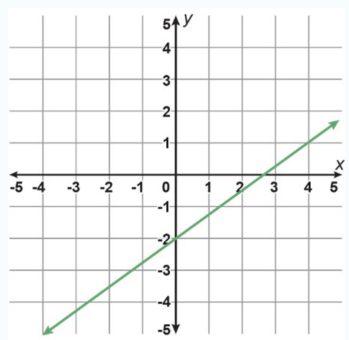
Point-slope form can be written using the slope and any point on the line.

3. Compare the equation and the graph of the line. What do you notice about the signs of the coordinates of the green point used to write the equation compared to the values in the equation?
4. Katrina wrote a different equation for the same line in point-slope form using the y-intercept, $(0,-2)$. Her equation is shown.

$$(y+2)=\frac{3}{4}(x-0)$$

   a. Explain why the expression $(y+2)$ is on the left side of the equation when the y-value of the point used is -2.
   b. Discuss with your partner how the equation $(y+2)=\frac{3}{4}(x-0)$ could be rewritten in slope-intercept form.
   c. Rewrite the equation $(y+2)=\frac{3}{4}(x-0)$ in slope-intercept form by isolating the variable y.
   d. Rewrite the original point-slope equation, $(y-1)=\frac{3}{4}(x-4)$, in slope-intercept form.
   e. What do you notice about the 2 equations you just generated?

The third form for the equation of a line is called *standard form.*

Standard Form	$3x-4y=8$
$Ax+By=C$ where A and B cannot both be zero	

5. Rewrite the equation $3x-4y=8$ in slope-intercept form by isolating the variable y.
6. Use the equation $f(x)=\frac{3}{4}x-4$ to complete the following.
   a. What is the form of the linear equation given?
   b. Rewrite the equation in standard form.

Possible Responses

1. No written response required. Sample discussions: The graphs are all the same. All the equations contain the numbers 3 and 4. The equations must be equivalent if their graphs are the same
2. Answers will vary. Sample response: This form is called *slope-intercept form* because the m represents the slope and the b represents the y-intercept
3. Answers will vary. Sample response: I notice that in the equation the values are negative, but the values are positive in the coordinates of the point. In this form, it looks like coordinates are subtracted. The equation uses a point on the line and the slope, so its name makes sense.
4.
   a. Answers will vary. Sample response: The point-slope formula subtracts the y-value on the left side. Substituting the y-intercept into the formula results in a double negative, which becomes positive.
   b. Students are not expected to write a response. Sample discussion: The fraction would be distributed first, and then, I would subtract 2 from both sides

c. $y+2=\frac{3}{4}(x-0)$
$y+2=\frac{3}{4}x-0$
$y=\frac{3}{4}x-2$

d. $y-1=\frac{3}{4}(x-4)$
$y-1=\frac{3}{4}x-3$
$y=\frac{3}{4}x-2$

e. Answers will vary. Sample response: Both equations have the same slope and y-intercept, so they are equivalent.

5. $3x-4y=8$
$-4y=-3x+8$
$y=\frac{3}{4}x-2$

6. 62.5%
a. Slope-intercept form
b. $y=\frac{2}{5}x-4$
$-\frac{2}{5}x+y=-4$ or equivalent in the form $Ax+By=C$

Anticipated Misconceptions

Some students may struggle with converting from slope-intercept form to standard form. Work through this process with students to remind them that it is another form of equivalent equations and keeping the sides equivalent by completing the same steps on each side.

Some students may find point-slope form challenging because of the signs. Display the general form, $(y-y_1)=m(x-x_1)$, to show students that the subtraction signs are a part of the equation and that substituting a negative number reverses the sign. Consider making an anchor chart of the 3 forms of the equation of a line for students to refer to in subsequent lessons and units.

Activity Synthesis

Highlight that each form of the equation of a line is formed by rearranging the formula, which is similar to rearranging formulas and equations students worked with in the previous lesson. Remind students that rearranging equations is important because certain forms are more useful at times than others depending on the information provided or sought, such as the slope, y-intercept, or points:

COLLABORATIVE ACTIVITY | EQUIVALENT FORMS *10–12 minutes*

Instructional Routines

- MLR8: Discussion Supports
- Take Turns

Students compare various equations given in all different forms and collaboratively practice rearranging equations to determine which equations are equivalent (MPS.RC.1).

LAUNCH

Arrange students in groups of 2. Give them 7–9 min. to collaboratively complete this activity. To complete problem 1, consider having students use the instructional routine Take Turns, in which they take turns rearranging an equation and sharing their thought process aloud. The listening partner asks clarifying questions, presses for more details, and determines whether they agree or disagree with their partner's work, and then they swap roles.

Support for English Language Learners

Speaking: MLR 8 Discussion Supports. Provide sentence frames to support students with explaining their thinking when making matches. For example, "I noticed that ________, so I ________" or "________ is in ________________ form because __________, so ________ must be the ____________ of the line." When students share their answers with a partner, prompt them to rehearse what they will say when they share with the full group. Rehearsing provides opportunities to clarify their thinking.

Design Principle(s): Optimize output (for explanation)

Support for Students with Disabilities

Conceptual Processing: Visual Aids. Create an anchor chart of the different forms of the equation of a line with the example graph explored in the previous activity (or display the activity on screen), which aids students who benefit from multiple pathways for language processing.

STUDENT ACTIVITY

Twelve linear equations are given in various forms.

$4x - 6y = 24$	$-4x - 6y = 24$	$-6x + 4y = 24$	$6x + 4y = 24$
$y = -\frac{2}{3}x - 4$	$f(x) = -\frac{2}{3}x - 6$	$y = \frac{3}{2}x + 6$	$f(x) = \frac{2}{3}x - 4$
$(y + 6) = -\frac{3}{2}(x - 8)$		$(y - 6) = \frac{3}{2}x$	
$(y + 6) = \frac{2}{3}(x + 3)$		$(y - 6) = -\frac{2}{3}(x + 15)$	

1. Using the equations, work with your partner to complete the table so that each row contains a set of equivalent equations, one written in each form. Not all equations will be used.

Equivalent Equations	Standard Form	Slope-Intercept Form	Point-Slope Form
Set 1			
Set 2			
Set 3			

Possible Responses

1.

Equivalent Equations	Standard Form	Slope-Intercept Form	Point-Slope Form
Set 1	$-6x+4y=24$	$y=\frac{3}{2}x+6$	$(y-6)=\frac{3}{2}x$
Set 2	$-4x-6y=24$	$y=-\frac{2}{3}x-4$	$(y-6)=-\frac{2}{3}(y+15)$
Set 3	$4x-6y=24$	$f(x)=\frac{2}{3}x-4$	$(y+6)=\frac{2}{3}(y+3)$

Anticipated Misconceptions

Some students who wish to rewrite the equations may be unsure where to begin. Have them rewrite equations to slope-intercept form like the previous activity to see the connections to equivalent equations.

Some students may struggle to determine a place to get started. Ask them which forms of an equation give the same information about the line and how that could be used to get started making matches.

Activity Synthesis

Select 3–4 students or pairs to share their responses and their methods for determining a set of equivalent equations. Consider asking the following questions during the whole-group discussion:

- "Did anyone use slope to help determine equivalent equations? If so, how?" (Both slope-intercept form and point-slope form reveal the slope of the line, so I looked for equations that have the same slope.)
- "How did you determine which equations in standard form matched other equivalent forms?" (Sample responses: I rewrote equations in slope-intercept form to standard form and then determined a quantity to multiply all the terms by to find the equivalent standard form. I determined a point on the line from point-slope form and then substituted the values of x and y into the standard form equations to determine which was true for that ordered pair.)

If no students share the preceding strategies, consider highlighting each as a strategy during the Lesson Synthesis.

LESSON SYNTHESIS

To help students consolidate their work in this lesson, review the Student Lesson Summary with the class. Consider providing students with the following example: "Transform the equation $y-3=2(x-1)$ into equivalent forms of slope-intercept and standard forms." To encourage discussion, ask students the following questions.

- "What information do we know about the line from the given form?" (The slope is 2, and $(1, 3)$ is a point on the line.)
- "Which form do you want to rearrange to first?" (Answers will vary.)
- "Can you determine equivalent equations without rearranging them?" (I could take the given information and use it in a different form to find the equation. For example, I could substitute the slope and the ordered pair into the slope-intercept form of an equation to find the value of b.)
- "When might it be useful to rearrange an equation into a different form?" (Other forms may be useful for finding the slope, matching the equation to a graph, or determining a point on the line).

This lesson establishes the foundation to see that understanding the structure and connections across equal representations provides deeper insights to equations and their graphs

Student Lesson Summary

Previous courses introduced the *slope-intercept form* for the equation of a line. The slope-intercept form of a line is $y = mx + b$, where m represents the *slope* of the line and b represents the *y-intercept*.

The **slope** of a line is the ratio of the change in the vertical direction to change in the horizontal direction, often expressed as $\frac{\text{change in } y}{\text{change in } x}$, or $\frac{\triangle y}{\triangle x}$.

The ***y*-intercept** is the value of y at the point where a line or graph intersects the y-axis. The value of x is 0 at this point.

In this lesson, 2 additional forms of linear equations were introduced.

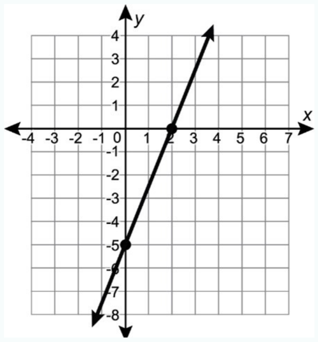
- The point-slope form of the equation of a line is $(y - y_1) = m(x - x_1)$, where m is the slope and (x_1, y_1) is any point on the line.
- The *standard form* of the equation of a line is $Ax + By = C$, where A and B cannot both be zero.

By using key features revealed in the form of the linear equation given, along with properties of operations and properties of equality, linear equations can be rewritten in other equivalent forms

COOL DOWN | THREE EQUATIONS AND A GRAPH *5 minutes*

STUDENT ACTIVITY

The graph of a linear equation is shown



1. Jarissa stated that the graph represents the equation $-5x + 2y = -10$. Rewrite Jarissa's equation in slope-intercept form.
2. Michael said the graph represents the equation $(y + 5) = \frac{5}{2}(x - 0)$. Rewrite Michael's equation in standard form.
3. Use your answers to problems 1 and 2 to explain which equations are equivalent.

Possible Responses

1. $y = \frac{5}{2}x - 5$
2. $y = \frac{5}{2}x - 5$
3. Answers will vary. Sample response: The equations $-5x+2y=-10$, $(y+5)=\frac{5}{2}(x\text{-}0)$, and $y=\frac{5}{2}x-5$ are all equivalent and represent the same line shown on the graph

PRACTICE PROBLEMS

PROBLEM 1

Use the equation $(y - 7) = -\frac{1}{3}(x + 2)$ to complete the following.

a. What is the form of the linear equation given?
b. Rewrite the equation in standard form.

Possible Solutions

a. Point-slope form

b. $(y-7) = -\frac{1}{3}(x+2)$

$y-7 = -\frac{1}{3}x - \frac{2}{3}$

$y = -\frac{1}{3}x - \frac{2}{3} + 7$

$\frac{1}{3}x + y = \frac{19}{3}$ or equivalent in the form $Ax + By = C$

PROBLEM 2

Use the equation $6x - 9y = -12$ to complete the following.

a. What is the form of the linear equation given?

b. Rewrite the equation in slope-intercept form.

Possible Solution

a. Standard form

b. $6x - 9y = -12$

$-9y = -6x - 12$

$y = \frac{2}{3}x + \frac{4}{3}$

PROBLEM 3

Select **all** of the equations that are equivalent to $(y+3) = -\frac{1}{4}x$.

- ☐ $y = -\frac{1}{4}x$
- ☐ $y = -\frac{1}{4}x - 3$
- ☐ $(y-3) = -\frac{1}{4}(x-1)$
- ☐ $x + 4y = 3$
- ☐ $x + 4y = -12$

Possible Solutions

- ☐ $y = -\frac{1}{4}x$
- ✓ $y = -\frac{1}{4}x - 3$
- ☐ $(y-3) = -\frac{1}{4}(x-1)$
- ☐ $x + 4y = 3$
- ✓ $x + 4y = -12$

PROBLEM 4

Is the equation $(y-5) = \frac{1}{2}(x+4)$ equivalent to $y = \frac{1}{2}x + 7$? Explain your thinking..

Possible Solution

Answers will vary. Sample response: Yes, they are equivalent equations. When the first equation is rewritten in slope-intercept form, it matches the second equation

PROBLEM 5

Review Problem

A car traveled 180 miles (mi.) at a constant rate.

a. Complete the table to show the rate at which the car was traveling if it completed the same distance in each number of hours (hr.)

b. Write an equation that would make it easy to find the rate at which the car was traveling in miles per hour (mph), r, if it traveled for t hr.

Travel Time (hr.)	Rate of Travel (mph)
5	
4.5	
3	
2.25	

Possible Solutions

Each card needs 12π or about 37.7 cm of ribbon. She has enough for 4 cards since $180 \div 37.7 \approx 4.77$.

a.

Travel Time (hr.)	Rate of Travel (mph)
5	36
4.5	40
3	60
2.25	80

b. $r = \frac{180}{t}$

PROBLEM 6

Review Problem

A group of 280 elementary school students and 40 adults are going on a field trip. They are planning to use two different types of buses to get to the destination. The first type of bus holds 50 people and the second type of bus holds 56 people.

Andre says that 3 of the first type of bus and 3 of the second type of bus will hold all of the students and adults going on the field trip. Is Andre correct? Explain your reasoning

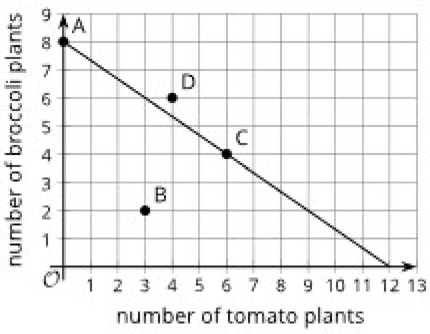
Possible Solutions

Andre is not correct because 320 people need to get on the buses: $3 \cdot 50 + 3 \cdot 56 = 318$, which is less than 320.

PROBLEM 7

Review Problem

To grow properly, each tomato plant needs 1.5 square feet (sq. ft.) of soil and each broccoli plant needs 2.25 sq. ft. of soil. The graph shows the different combinations of broccoli and tomato plants in an 18 sq. ft. plot of soil.



Match each point to the statement that describes it.

Point
a. Point A
b. Point B
c. Point C
d. Point D

Statement
The soil is fully used when 6 tomato plants and 4 broccoli plants are planted.
Only broccoli was planted, but the plot is fully used and all plants can grow properly.
After 3 tomato plants and 2 broccoli plants were planted, there is still extra space in the plot.
With 4 tomato plants and 6 broccoli plants planted, the plot is overcrowded.

Possible Solutions

- **a.** Point A – Only broccoli was planted, but the plot is fully used and all plants can grow properly.
- **b.** Point B – After 3 tomato plants and 2 broccoli plants were planted, there is still extra space in the plot.
- **c.** Point C – The soil is fully used when 6 tomato plants and 4 broccoli plants are planted.
- **d.** Point D – With 4 tomato plants and 6 broccoli plants planted, the plot is overcrowded.

REFLECTION AND NOTES

LESSON 3
ADDING AND SUBTRACTING COMPLEX NUMBERS

LEARNING TARGETS

- *I can define a complex number.*
- *I can add and subtract complex numbers.*

ALIGNMENT

Addressing

- **A2P.NR.1.1** Understand that there is an imaginary unit i such that $i^2 = -1$ and explain the structure of a complex number as $a + bi$, where a and b are real.
- **A2P.NR.1.2** Add, subtract, and multiply complex numbers.

LESSON PREPARATION

Required Materials

- Cool Down

LESSON NARRATIVE

Students define the structure of complex numbers as a part of the number system hierarchy and focus on the structure of complex numbers as a real term plus an imaginary term to calculate sums and differences.

This lesson establishes the foundation for understanding that complex numbers follow the same basic rules for addition and subtraction as other number systems that are familiar to students, such as integers, polynomials, and radicals (MPS.SP.1). In other words, the sum and difference of 2 complex numbers will result in a complex number. Students recognize this by using usual arithmetic and the fact that $i^2 = -1$, which they explored in the previous lesson, to write sums and differences in the form $a + bi$, where a and bi are real numbers. They add and subtract complex numbers (MPS.PS.1), and in the next lesson, students multiply complex numbers. However, it is beyond the scope of this course to consider quotients of complex numbers.

WARM-UP | MATH TALK: TELESCOPING SUMS *5 minutes*

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

This Math Talk elicits strategies and the understanding students have for adding and subtracting integers, which help them develop fluency and are helpful later in this lesson when students combine like terms to express the sums and differences of complex numbers in the form $a + bi$, where a and b are real numbers.

Students notice and make use of structure (MPS.SP.1) because each sum includes several pairs of opposites that sum to 0. Being able to compute efficiently by choosing which parts of an expression to evaluate first will be helpful as students develop fluency with complex number arithmetic.

LAUNCH

Display 1 problem at a time. Give students 30 seconds (sec.) of quiet think time for each problem, and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk, and follow up with a whole-class discussion.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, “First, I ______ because . . .” or “I noticed ______ so I . . . .” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

STUDENT ACTIVITY

1. Find the value of these expressions mentally.
   $2 - 2 + 20 - 20 + 200 - 200$
   $100 - 50 + 10 - 10 + 50 - 100$
   $3 + 2 + 1 + 0 - 1 - 2 - 3$
   $1 + 2 + 4 + 8 + 16 + 32 - 16 - 8 - 4 - 2 - 1$

Possible Responses

$0, 0, 0, 32$

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses. To involve more students in the conversation, consider asking the following questions:

- “Who can restate ______’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in another way?”

- “Does anyone want to add on to ______’s strategy?”
- “Do you agree or disagree? Why?”

GUIDED ACTIVITY | THE COMPLEX NUMBER SYSTEM *12–15 minutes*

Instructional Routines

- MLR5: Co-Craft Questions
- Notice and Wonder
- Poll the Class

Students reason about the difference between real and complex numbers by generalizing their structure (MPS. SP.1) and begin by using a Venn diagram to visualize the relationship between number sets. They then distinguish between complex numbers and real numbers and classify real numbers according to the subsets they belong to.

LAUNCH

Arrange students in groups of 2. Introduce the concept of a *complex number* using the explanation provided in the Student Edition. Call on a few students to restate their understanding of complex numbers, including what makes them complex and what it means for a number to be purely imaginary. Annotate student ideas as they are shared. Give students 1–2 minutes (min.) to complete problem 1 on their own, and then, to check for understanding, ask randomly selected students to each share where they placed 1 of the values in the table.

Display the Venn diagram from the activity for all to see. Ask students to think of at least 1 thing they notice and at least 1 thing they wonder about the image. Give students 1 min. of quiet think time and 1 min. to discuss the things they noticed and wondered with their partner. Facilitate a quick whole-class discussion to get students familiar with the Venn diagram and how to interpret it. Give students 3–4 min. to complete the activity with their partner. Encourage them to discuss each value in problem 2 and, if a disagreement arises, to work together to reach an agreement. Listen as students discuss problem 3 to identify key ideas to highlight during the whole-class discussion.

Support for English Language Learners

Conversing, Writing: MLR5 Co-Craft Questions. Use this routine to spark students’ curiosity about the number system hierarchy. Display only the Venn Diagram of the complex number system, and ask students to write down possible mathematical questions that could be asked about the image. Invite students to compare their questions before revealing problem 2. Listen for and amplify any questions involving the relationship between the different number types. This will help students create the language of mathematical questions before feeling the pressure to produce solutions.

Design Principle(s): Maximize meta-awareness; Support sense-making

Support for Students with Disabilities

Representation: Internalize Comprehension. Use color-coding of the different number types to highlight connections in complex numbers. For example, highlight real numbers in 1 color and imaginary numbers in a different color. This way students will see that complex numbers may be a combination or real and imaginary parts.

Supports accessibility for: Visual-spatial processing

STUDENT ACTIVITY

A *complex number* is a number that can be written in the form $a + bi$, where a and b are real numbers. In the form $a + bi$, a is considered the **real** part of the complex number, and bi is the **imaginary** part of the complex number. Complex numbers where $a = 0$ are often referred to as *pure imaginary numbers*.

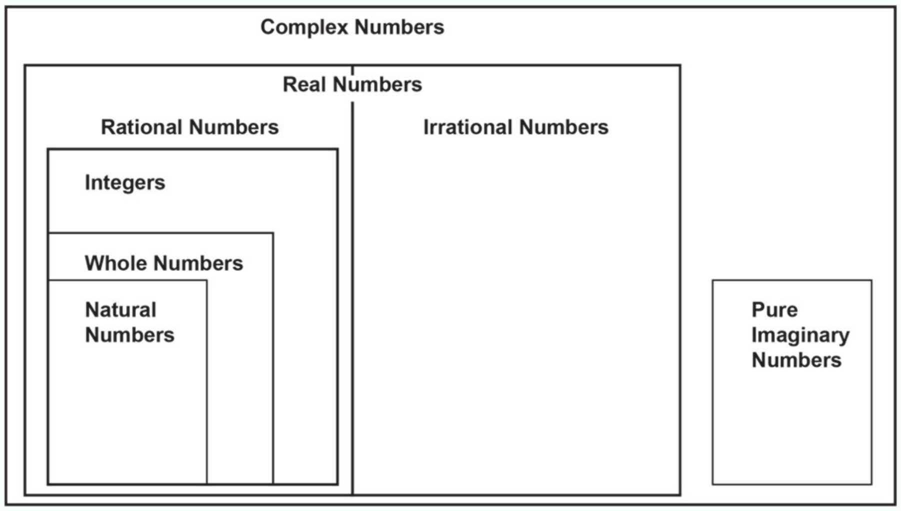
1. A list of numbers is shown.

$$3, \frac{\sqrt[3]{2}}{2}, -2i, 4 + \sqrt{6}i, -\frac{1}{6}, \pi - i, \frac{1}{2}(1 - \sqrt{-4}), \sqrt[3]{-8}, 5i^2, \frac{i}{3}$$

Sort the numbers into the categories shown in the table.

Complex Numbers $(a + bi)$		
Real Numbers $(b = 0)$	Pure Imaginary Numbers $(a = 0, b \neq 0)$	Other Complex Numbers $(a \neq 0, b \neq 0)$

The number system includes categories, or subsets, of numbers. With the introduction of additional number types, the system grows to represent the unique characteristics of each number. A Venn diagram representing the relationship between different number types within the complex number system is shown.



2. Classify each real number expression by writing them in the appropriate sections of the Venn diagram.

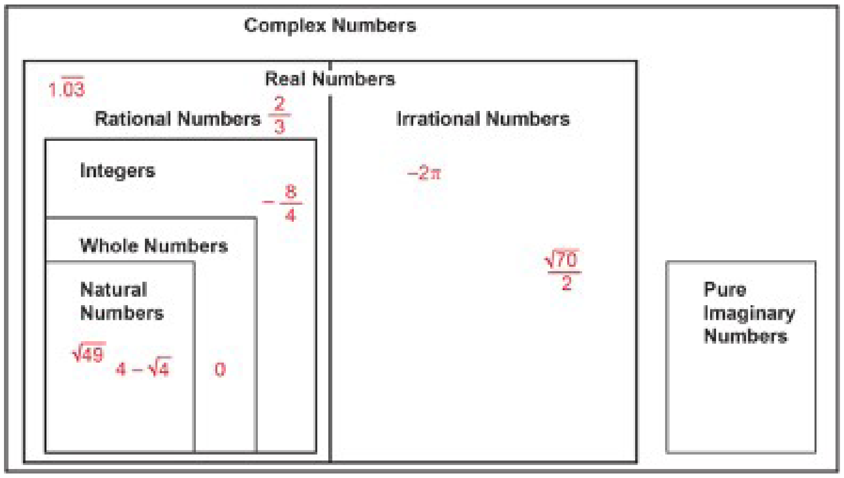
$$\sqrt{49}, \frac{2}{3}, -2\pi, -\frac{8}{4}, 0, \frac{\quad}{2}, 4 - \sqrt{4}, 1.\overline{03}$$

3. Two new subsets of numbers have been introduced in this unit. Discuss with your partner whether an imaginary number is also a complex number.

Possible Responses

1.

Complex Numbers $(a + bi)$		
Real Numbers $(b = 0)$	Pure Imaginary Numbers $(a = 0, b \neq 0)$	Other Complex Numbers $(a \neq 0, b \neq 0)$
$3, \frac{\sqrt[3]{3}}{2}, -\frac{1}{6} \sqrt[3]{-8}, 5i^2$	$-2i, \frac{i}{3}$	$4 + \sqrt{6}i, \pi - i, \frac{1}{2}(1 - \sqrt{-4})$

2. 

3. No written response required. Sample discussion: An imaginary number is also a complex number because it is part of the larger complex number hierarchy in the Venn diagram.

Activity Synthesis

Facilitate a whole-class discussion by inviting students to share their responses to problem 2 while placing the values in the Venn diagram as students identify where each should go. It is important to discuss that some of the values in the Venn diagram are expressions that could be rewritten differently but can still be classified as a number type within the complex number system. If students need additional practice understanding the complex number system hierarchy, consider asking them to identify all the number types (and placement within the Venn diagram) of each of the following numbers:

- $-\sqrt{16}$ (real, rational, integer)
- $4i+3$ (complex)
- $4i^2+9$ (real, rational, integer, whole, natural)
- $3i$ (pure imaginary)

Poll the Class to determine whether students think an imaginary number is also a complex number. (It is because it is a subset of complex numbers. Purely real numbers are also considered complex numbers because they are a subset of this number system.)

EXPLORATION ACTIVITY | ADDING AND SUBTRACTING COMPLEX NUMBERS *8–12 minutes*

Instructional Routines

- MLR1: Stronger and Clearer Each Time

This activity extends the work of this lesson to operations with complex numbers, and students make connections between combining like terms of polynomials and addition and subtraction of complex numbers (MPS.C.1).

LAUNCH

Arrange students in groups of 2. Give students 2–3 min. to complete the first 2 problems individually. Then, give students 1–2 min. to compare their responses and their explanations to problem 2 with their partner. Ask students to discuss any similarities and differences in their explanations and come to agreement on which explanation they would like to share with the class. Monitor for students who identify the distributive property and combining like terms as their strategies. Invite 2–3 groups to share their discussion summaries with the class before releasing students for another 2–3 min. to complete problem 3 collaboratively.

Support for English Language Learners

Writing, Conversing: MLR1 Stronger and Clearer Each Time. Use this routine to help students improve their writing by providing them with multiple opportunities to clarify their explanations through conversation. Give students time to meet with2–3partners to share their response to the question "How did Louis add and subtract the 2 expressions?" Students should first check to see if they agree with each other about what strategies Louis used. Provide listeners with prompts for feedback that will help their partner add detail to strengthen and clarify their ideas. For example, students can ask their partner, "Did Louis rewrite the expression in any way before adding or subtracting?" or "What do you notice about which parts of the expressions he combined?" Next, provide students with time to revise their initial draft based on feedback from their peers. This will help students produce clear explanations for how to add and subtract complex numbers.

Design Principle(s): Support sense-making; Optimize output (for explanation)

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to another with which students have experienced success. For example, write complex numbers with variables instead of i to allow students to see different parts of the expressions and like terms that can be added and subtracted.

Supports accessibility for: Social-emotional skills; Conceptual processing

STUDENT ACTIVITY

1. For each complex number, place a box around the real part, and circle the imaginary part.
   $5 + 2i$ $2 - 3i$ $-5i + 23$ $10 + 2i$
2. Louis was asked to add and subtract 2 complex numbers, $2 + 5i$ and $3 - 8i$. His work is shown.

Add	$(2 + 5i) + (3 - 8i) = 5 - 3i$
Subtract	$(2 + 5i) - (3 - 8i) = 2 + 5i - 3 + 8i = -1 + 13i$

   With your partner, explain what Louis did to add and subtract the 2 expressions.
3. Perform the given operation for the complex numbers.
   a. $(3 + 5i) + (7 - 2i)$
   b. $(3 - 8i) - (6 + 12i)$

Possible Responses

1. $\boxed{5} + \textcircled{2i}$ $\boxed{2} - \textcircled{3i}$ $\textcircled{-5i} + \boxed{23}$ $\boxed{10} + \textcircled{2i}$
2. Answers will vary. Sample response: For addition, Louis combined the like parts of the expressions. For subtraction, Louis distributed a negative sign (or $a - 1$) to the second set of parentheses and then combined like terms.
3.
   a. $10 + 3i$
   b. $3 - 8i - 6 - 12i$
      $-3 - 20i$

Anticipated Misconceptions

Students may forget to distribute the -1 in problem 3b. Remind students of the distributive property and that $(3-8i)-(6+12i)$ is the same as $(3-8i)+(-1)(6+12i)$.

Activity Synthesis

To wrap up the activity, invite 2–3 groups to share their solutions to problems 3a and 3b. Amplify student explanations that include mathematical terminology such as *real part*, *imaginary part*, or *distributive property*. Invite the class to agree or disagree with the methods used and to come to a consensus on the final answers.

COLLABORATIVE ACTIVITY | ADDING AND SUBTRACTING COMPLEX NUMBERS MAZE *5–8 minutes*

Instructional Routines

- MLR8: Discussion Supports
- Take Turns

Students apply their understanding of sums and differences of complex numbers to complete a maze.

LAUNCH

Determine which instructional structure is best for your students based on their understanding at this point in instruction.

- If implementing this activity as individual practice, give students 4–5 min. to complete the maze before comparing their solution methods to their peers'. Consider embedding checkpoints, such as after solving the first 3 boxes, and then check to make sure they agree before moving on. Encourage students to use Take Turns for each step of the maze.
- If implementing this activity collaboratively, arrange students in groups of 2. Give students 1 min. of quiet think time to determine strategies they will use to complete the maze. Give students 4–5 min. to work through the maze with their partners, encouraging students to use Take Turns if implementing in this way. Monitor students' discussions to identify strategies to highlight during the whole-class discussion. Observe sums or differences that students seem to struggle with, and clarify those during the activity synthesis.
- If implementing this activity in teacher-supported groupings, consider bringing some groups of students that needed additional support in the previous activities to a teacher station. Provide scaffolding strategies, such as circling the real part and drawing a box around the imaginary part of complex numbers or asking guiding questions such as "Which terms are alike?", "Is this a sum or difference calculation?", or "Should we distribute the sign in front?"

In any of these approaches, students should be encouraged to notice patterns and structure in repeated calculations and look for possible generalizations and shortcuts (MPS.SP.1)

Support for English Language Learners

Conversing: MLR8 Discussion Supports. Students should take turns finding a match in the maze and explaining their reasoning to their partner. Display the following sentence frame for all to see: "I noticed ______, so I matched . . . ." Encourage students to challenge each other when they disagree. This will help students clarify their reasoning about even and odd functions.

Design Principle(s): Support sense-making; Maximize meta-awareness

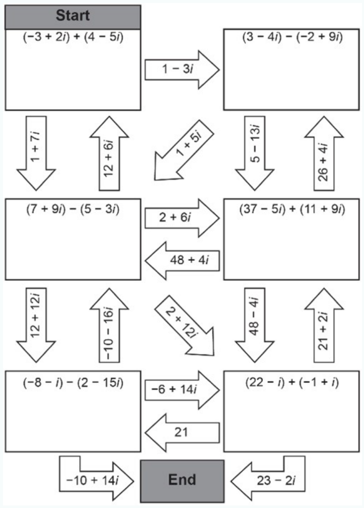
Support for Students with Disabilities

Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer questions. For example, give students a portion of the maze to complete, and then introduce remaining questions if time allows.

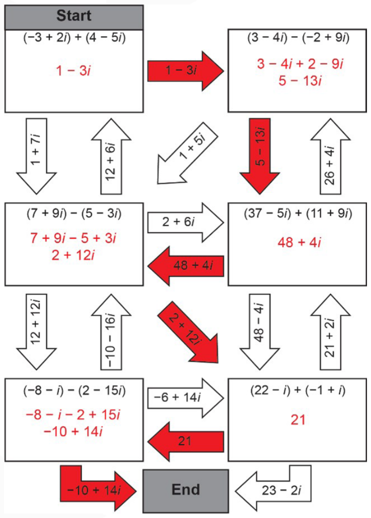
Supports accessibility for: Conceptual processing; Organization

STUDENT ACTIVITY

Add or subtract each complex expression. Shade in the correct path to complete the maze from start to end.



Possible Responses



Activity Synthesis

Display 2–3 previously identified sums and differences that students struggled with during the activity. Invite students to brainstorm any hurdles in completing the operations with complex numbers they may have had and any strategies that can be used to overcome the hurdle. If most students did not struggle with the activity, consider simply displaying the completed maze and asking students if they have any problems they want to discuss or have questions about.

LESSON SYNTHESIS

Display the prompt, "How are complex numbers and real numbers similar? How are they different?" Ask students to reflect on their learning to answer these questions. After 2 min. of quiet work time, invite students to share their reflections. Write student responses on the board, highlighting student reflections that include the idea that complex numbers can be combined with similar terms (real and imaginary parts). Refer to the Student Lesson Summary for examples to review the work of the lesson.

Student Lesson Summary

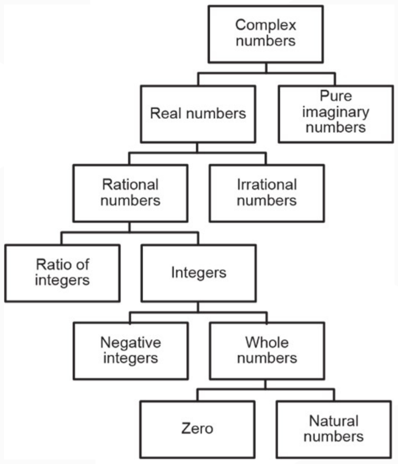
When a real number and an imaginary number are combined, the result is a *complex number*.

> A **complex number** is a number in the complex plane. It can be written as $a + bi$, where a and b are real numbers and $i^2 = -1$.

A complex number has 2 parts: the real part, a, and the imaginary part, bi.

The number system can be organized using a Venn diagram as shown in the lesson or as a hierarchy where each subset of numbers is shown below the broader number type. The complex number system's hierarchy is shown in the image.

Notice that real numbers and pure imaginary numbers are both subsets of complex numbers.



- Real numbers are complex numbers where $b = 0$.
- Pure imaginary numbers are complex numbers where $a = 0$.

The process of adding and subtracting complex numbers is similar to that of adding and subtracting polynomials. With polynomials, like terms are combined. With complex numbers, like parts are combined.

The sum and difference of the complex numbers $2 + 3i$ and $4 + 5i$ are shown.

$$(2 + 3i) + (4 + 5i) = (2 + 4) + (3i + 5i) = 6 + 8i$$
$$(2 + 3i) - (4 + 5i) = (2 - 4) + (3i - 5i) = -2 - 2i$$

In general, the sum (or difference) of complex numbers results in the sum (or difference) of the coefficients of the real parts and the coefficients of the imaginary parts of the complex numbers.

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$
$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

COOL DOWN | COMPLEX OPERATIONS *5 minutes*

STUDENT ACTIVITY

1. Add or subtract each complex expression.
   a. $(5 + 3i) + (6 - 8i)$ b. $(3 - 2i) - (2 - 4i)$ c. $6 + (4 - 3i)$

Possible Responses

a. $11 - 5i$ b. $1 + 2i$ c. $10 - 3i$

PRACTICE PROBLEMS

PROBLEM 1

Which expression is equivalent to $(3+9i)-(5-3i)$?

A. $-2-12i$ B. $-2+12i$ C. $15+27i$ D. $15-27i$

Possible Solutions

B

PROBLEM 2

What are a and b when you write $\sqrt{-16}$ in the form $a+bi$, where a and bi are real numbers?

A. $a=0,\ b=-4$ B. $a=0,\ b=4$ C. $a=-4,\ b=0$ D. $a=4,\ b=0$

Possible Solutions

B

PROBLEM 3

Fill in the boxes to make a true statement:

$$(\square-3i)-(15+\square i)=7-12i$$

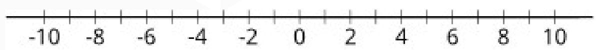
Possible Solutions

22 and 9

PROBLEM 4

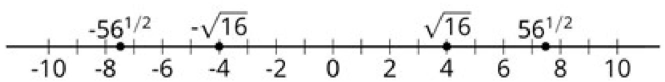
Review Problem

Plot each number on the real number line, or explain why the number is not on the real number line.



a. $\sqrt{16}$
b. $-\sqrt{16}$
c. $\sqrt{-16}$
d. $56^{\frac{1}{2}}$
e. $-56^{\frac{1}{2}}$
f. $(-56)^{\frac{1}{2}}$

Possible Solutions



Sample response: $\sqrt{-16}$ and $(-56)^{\frac{1}{2}}$ do not appear on the number line because there is no real number you can square and get -16 or -56.

PROBLEM 5

Review Problem

Which expression is equivalent to $\sqrt{-4}$?

A. $-2i$ B. $-4i$ C. $2i$ D. $4i$

Possible Solutions

C

LESSON 1
REVISITING EQUATIONS OF LINES

LEARNING TARGET

- *I can write the equation of a line in slope-intercept form when given the slope and point on a line.*

ALIGNMENT

Building Toward

- **GS.PAFR.2.2** Analyze slopes of lines to determine whether lines are parallel, perpendicular, or neither.
- **GS.PAFR.2.3** Determine the equation of a line passing through a given point that is parallel or perpendicular to a given line.

LESSON PREPARATION

Required Materials

- Cool Down

LESSON NARRATIVE

In 8th grade, students used similar triangles to explain why the slope m is the same between any 2 distinct points on a nonvertical line in the coordinate plane. Using this understanding, students derived the equation $y = mx + b$ for a line intercepting the vertical axis at b. In this lesson, students use the slope-intercept form of a line to write an equation when given the slope and a point on the line. Although equations of lines can be written in different forms, this course focuses on only using the slope-intercept form when writing an equation of a line because the point-slope and standard forms of lines are not introduced until Algebra 1.

Slope calculations are an important part of this lesson, so students begin with a Warm-Up designed to help them recall this concept from the previous course. Then, they use the definition of *slope*, represented as $m = \frac{\Delta y}{\Delta x}$, to build an equation of a line when given a point and the slope. In the third activity, students practice writing and interpreting equations of lines in slope-intercept form.

Although students may see equations that are written in point-slope form within the lesson, they are not referred to as such, and students are not expected to produce a line in this form. Instead, the lesson relies on the slope formula and the slope-intercept form of the equation of a line to build an understanding of equations of lines that they will use in subsequent lessons to explore the relationships between the slopes of parallel and perpendicular lines. This leads to writing equations of lines passing through a given point that is parallel or perpendicular to a given line (GS.PAFR.2.3). This lesson acts as a foundation for the lessons that follow.

Throughout this lesson, students engage in mathematical discourse when they explain their reasoning when writing an equation of a line given the slope and a point on the line (MPS.RC.1).

WARM-UP | REMEMBERING SLOPE *5–7 minutes*

Instructional Routines

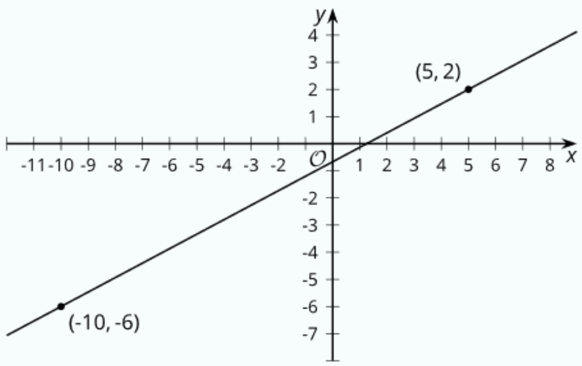
- Think-Pair-Share

This Warm-Up reviews the concept of slope, which students learned in 8th grade. Review students' work to determine whether the concept of slope needs to be revisited prior to the Exploration Activity.

LAUNCH

Arrange students in groups of 2, and tell them that there are many possible answers for the problem. After 1 minute (min.) of quiet work time, ask students to compare their responses to their partner's and decide whether they are both correct, even if their responses are different. While students engage with their partners, monitor students' work for those who draw a slope triangle and for those who use a slope formula or equation. Follow up with a whole-class discussion.

STUDENT ACTIVITY



The slope of the line in the image is $\frac{8}{15}$. Explain how you know this is true.

Possible Responses

Answers will vary. Sample response: I drew a slope triangle. The vertical leg of the triangle is 8 units long, and the horizontal leg is 15 units long. Slope is calculated by dividing the vertical length by the horizontal length, so the slope is $\frac{8}{15}$.

Activity Synthesis

This discussion highlights the use of the expression $\frac{(2-(-6))}{(5-(-10))}$ to determine slope. Seeing the coordinates subtracted in this way will help students as they work through upcoming activities, and students may also write the equivalent expression $\frac{(-6-2)}{(-10-5)}$ when using the given points to demonstrate the slope is $\frac{8}{15}$.

Based on the observations of students' work and class discussion, differentiate the Warm-Up by including the following points about slope if needed:

- Ask students what *slope* means. Students may describe slope as the steepness of a line, the rate of change in a linear relationship, or "the change in y over the change in x." Remind them that one way to think about slope is that it is the quotient of the lengths of the legs of a slope triangle: $\text{vertical distance} \div \text{horizontal distance}$.

A right triangle drawn between any 2 points on a line will produce the same slope result because the slope triangles that are formed are similar.

- Invite a student who drew a slope triangle to share their work. Display the student's slope triangle, or draw one of your own. Ask students how they can calculate the lengths of the legs of the triangle. As students describe how to do so, label the legs $2-(-6)$ and $5-(-10)$
- Then, write out the slope as $\frac{(2-(-6))}{(5-(-10))}$. It is important that students see this expression in preparation for their work in the next activity.

If any students used a slope formula, ask them how the formula relates to this expression and whether the order of the numbers matters. ("The order must be consistent. Because we started with the 2 in the numerator, we have to start with the corresponding coordinate of the same point, 5, in the denominator.")

EXPLORATION ACTIVITY | BUILDING AN EQUATION FOR A LINE *12–15 minutes*

Instructional Routines

- MLR8: Discussion Supports

Students create an equation of a line given a slope and a point and explore how the slope formula they reviewed in the Warm-Up can be rearranged into slope-intercept form. This activity has students using the slope formula to write the equation of a line.

LAUNCH

Arrange students in groups of 2–3. Begin the activity by revisiting the slope-intercept form. Ask students the following questions:

- "What information does the slope-intercept form tell us about a line?" (The slope and y-intercept)
- "In the equation, $y=\frac{2}{3}x-8$, what is the slope? What is the y-intercept?" (The slope is $\frac{2}{3}$, and the y-intercept is -8.)
- "How can the equation $2x+4y=7$ be rewritten in slope-intercept form?" (Subtract $2x$ from both sides of the equation, and then divide both sides of the equation by 4.)

Provide students with 3–4 min. of collaborative work time to complete problem 1. Engage in a whole-class discussion for 3–4 min. to review problem 1. This can be a teacher-led process in which you use a think-aloud technique to share your thought process and reasoning, or it can be a student-led discussion in which selected students share their responses and justifications with the class.

Give students 2–3 min. of collaborative work time to complete problem 2 with their groups before bringing the class back together to discuss.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to help students produce statements explaining how they wrote the equation in slope-intercept form. Provide sentence frames such as, "First, I ______ because ______." and "I know the slope is ______ because ______."

Design Principle(s): Support sense-making; Optimize output (for explanation)

Support for Students with Disabilities

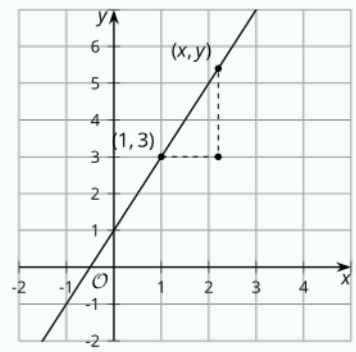
Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of *slope*, *y-intercept*, and *slope-intercept form*.

Supports accessibility for: Conceptual processing; Language

STUDENT ACTIVITY

Recall from 8th grade that equations representing linear relationships can be written in *slope-intercept form*.

- Slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.
- The slope, m, is the ratio of the change in the vertical direction to the change in the horizontal direction between 2 points, often expressed as $\frac{\Delta y}{\Delta x}$.



1. The graph of a linear equation is shown on the coordinate plane.
   - **a.** Write an equation that shows the slope between the points $(1, 3)$ and (x, y) is 2.
   - **b.** Consider the equation $y - 3 = 2\,(x - 1)$. How does it relate to the equation you wrote?
   - **c.** Rewrite the equation from part A in slope-intercept form.
2. Another linear equation can be represented by the equation $\frac{y-7}{x-5} = \frac{1}{2}$.
   - **a.** What point do you know is on this line?
   - **b.** What is the slope of the line?
   - **c.** Rewrite the equation in slope-intercept form.

Possible Responses

1. **a.** $\frac{y-3}{x-1} = 2$ or equivalent
   b. It is the same equation, but it was rearranged by multiplying both sides by $(x - 1)$.
   c. $y = 2x + 1$
2. **a.** $(5, 7)$
   b. $\frac{1}{2}$
   c. $y = \frac{1}{2}x + \frac{9}{2}$

Anticipated Misconceptions

If students struggle with the first question, then suggest they label the lengths of the legs of the slope triangle in the graph.

Activity Synthesis

This discussion ensures students understand the structure of slope-intercept form. Here are some questions for discussion:

- "How do you find the slope of a line? What does it tell you about the line?"
- "How can the slope of a line be used to write an equation of a line in slope-intercept form?"
- "What information is needed to write an equation of a line in slope-intercept form?"
- "How can you rearrange an equation into slope-intercept form?"

COLLABORATIVE ACTIVITY | EQUATIONS OF LINES *12–15 minutes*

Instructional Routines

- Poll the Class
- MLR3: Clarify, Critique, Correct

Students work in small groups to solve problems involving the equations of lines in slope-intercept form (MPS.RC.1) and practice writing and interpreting equations in slope-intercept form. Two strategies are introduced in problem 1, and then students can determine which strategy they want to use when writing equations in problem 2 (MPS.PS.1).

LAUNCH

Arrange students in groups of 2. Allow students the opportunity to work through this activity with their partners. Provide students with 4–5 min. of collaborative work time to complete problem 1. Engage the class in a discussion, and ask different pairs to share their responses to check for understanding before moving on. Give students another 4–5 min. of collaborative work time to complete the remainder of the activity before bringing the class together for a whole-class discussion.

Support for English Language Learners

Reading, Writing: MLR3 Clarify, Critique, Correct. Present a first draft of writing the equation of a line for problem 2a. Prompt discussion by asking, "What were the steps the author took?" Ask students to clarify and correct the work. Improved work should include simplifying the equation into slope-intercept form. This will help students understand the process of writing an equation of a line given a point and slope in slope-intercept form.

Design Principle(s): Maximize meta-awareness; Optimize output (for explanation)

Support for Students with Disabilities

Social-Emotional Functioning: Peer Tutors. Pair students with their previously identified peer tutors.

Supports accessibility for: Social-emotional skills

STUDENT ACTIVITY

1. Josiah and Claudia each wrote the equation of a line that passes through the point $(-7, 5)$ with slope 3 in slope-intercept form. Their work is shown.

Josiah's Work	Claudia's Work
$\frac{y-5}{x-(-7)} = 3$ $\frac{y-5}{x+7} = 3$ $y - 5 = 3(x + 7)$ $y - 5 = 3x + 21$ $y = 3x + 26$	$y = mx + b$ $5 = (3)(-7) + b$ $5 = -21 + b$ $26 = b$ $y = 3x + 26$

a. Discuss with your partner the method Josiah used to write the equation of the line in slope-intercept form. Summarize his method.

b. Discuss with your partner the method Claudia used to write the equation of the line in slope-intercept form. Summarize her method.

c. Use each of their methods to write the equation of a line that passes through the point $(6, 1)$ with slope $\frac{1}{2}$ in slope-intercept form.

Josiah's Work	Claudia's Work

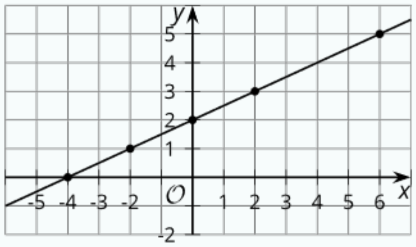
2. Write an equation of a line in slope-intercept form from each description.

   a. The line passing through point $(-2, 8)$ with slope $\frac{4}{5}$

   b. The line passing through point $(0, 7)$ with slope $-\frac{7}{3}$

   c. The line passing through point $\left(\frac{1}{2}, 0\right)$ with slope -1

   d. The line in the image



3. Determine the slope and a point that each line passes through using the structure of each equation.

   a. $\frac{y-5}{x+4} = \frac{3}{2}$

   b. $y = 5x - 2$

   c. $y = -2\left(x - \frac{5}{8}\right)$

Possible Responses

1. a. Answers will vary. Josiah set up the slope formula using points (x, y) and $(-7, 5)$. Then, he rewrote the equation in slope-intercept form.

   b. Answers will vary. Anneliese substituted the slope and the values $(-7, 5)$ in for (x, y) in the slope-intercept form and then solved for b. Then, she rewrote the slope-intercept form using the slope and y-intercept.

   c.

Josiah's Work	Claudia's Work
$\frac{y-1}{x-6} = \frac{1}{2}$	$y = mx + b$
$y - 1 = \frac{1}{2}(x - 6)$	$1 = \left(\frac{1}{2}\right)(6) + b$
$y - 1 = \frac{1}{2}x - 3$	$1 = 3 + b$
$y = \frac{1}{2}x - 2$	$-2 = \mathrm{b}$
	$y = \frac{1}{2}x - 2$

2. a. $y = \frac{4}{5}x + \frac{48}{5}$

   b. $y = \frac{7}{3}x + 7$

   c. $y = x + \frac{1}{2}$

   d. $y = \frac{1}{2}x + 2$

3. a. The line passes through $(-4, 5)$. The slope is $\frac{3}{2}$.

   b. The line passes through $(0, -2)$. The slope is 5.

   c. The line passes through $\left(\frac{5}{8}, 0\right)$. The slope is -2.

Anticipated Misconceptions

If students struggle with identifying the point that the line passes through in problem 3, then suggest that they look back to the first question. For lines with points that included the number 0, ask students, "How can those be rewritten so that the 0 doesn't appear? Do any of those forms look similar to the equations in problem 2?"

Activity Synthesis

Use the Poll the Class routine to review student responses for problems 2 and 3. Display each problem one part at a time, and have students record their responses on a mini whiteboard or a piece of paper and hold them up. Use students' responses to invite 2–3 students to explain the strategy that they used to determine their answers. Consider selecting students who identified different but equivalent equations.

LESSON SYNTHESIS

Display the example from the Student Lesson Summary, and review the Student Lesson Summary with the class to wrap up this lesson.

Consider having students determine the equation of the line using the given information if additional practice is needed. Call on a few students to share their process, selecting 1–2 students who used the slope formula and 1–2 students who used the slope-intercept form.

To involve more students in the conversation, consider asking the following questions:

- "Can anyone explain the steps in a different way?"
- "Does anyone want to add on to __________'s strategy?"
- "Do you agree or disagree with their strategy? Why?"

Remind students they can refer to the Lesson Summary when practicing on their own as a refresher of what they learned in class.

Student Lesson Summary

This lesson focuses on writing the equation of a line in slope intercept form, $y = mx + b$, where m is the *slope* of the line and b is the y-*intercept* of the line.

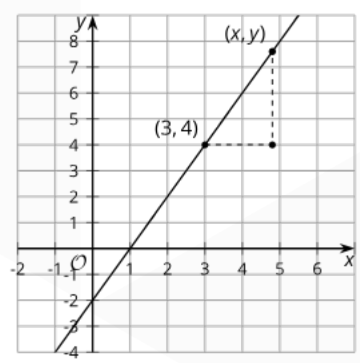
> The **slope** is the ratio of the change in the vertical direction (y-direction) to the change in the horizontal direction (x-direction), often expressed as $\frac{\Delta y}{\Delta x}$.

> The **y-intercept** is the value of y at the point where a line or graph intersects the y-axis. The value of x is 0 at this point.

There are an infinite number of points, (x, y), that satisfy the equation of a line.

The line shown on the coordinate plane can be defined as the set of points that includes the point $(3, 4)$ and has a slope of 2. The given information can be used to write an equation of the line.

- One method is using slope, $m = \frac{\Delta y}{\Delta x}$, read, "Slope is the change in y over the change in x." The points $(3, 4)$ and (x, y) can be used along with the slope to write the equation shown. This equation can then be rearranged into slope-intercept form.



$$\frac{y-4}{x-3} = 2$$
$$y - 4 = 2(x - 3)$$
$$y - 4 = 2x - 6$$
$$y = 2x - 2$$

- Another method that can be used to write an equation of a line that passes through point $(3, 4)$ with a slope of 2 is to substitute the (x, y) values of the known point and the slope into slope-intercept form to find the value of b. Then, use the values of m and b to write the equation.

$$y = mx + b$$
$$4 = 2(3) + b$$
$$4 = 6 + b$$
$$-2 = b$$
$$y = 2x - 2$$

Notice that both methods result in the same equation in slope-intercept form.

COOL DOWN | SAME SLOPE, DIFFERENT POINT *5 minutes*

STUDENT ACTIVITY

Consider the line represented by $y = \frac{2}{3}x - 10$. Write an equation representing a different line with the same slope that passes through the point $(3, 6)$.

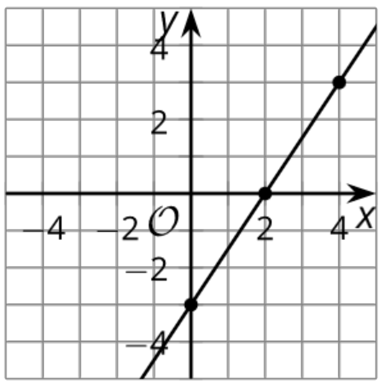
Possible Responses

$y = \frac{2}{3}x + 4$

PRACTICE PROBLEMS

PROBLEM 1

Select all of the equations that represent the graph shown.



- ☐ $3x - 2y = 6$
- ☐ $y = \frac{3}{2}x + 3$
- ☐ $y = \frac{3}{2}x - 3$
- ☐ $\frac{y-3}{x-4} = 3$
- ☐ $\frac{y-6}{x-2} = \frac{3}{2}$

Possible Solutions

- ✓ $3x - 2y = 6$
- ☐ $y = \frac{3}{2}x + 3$
- ✓ $y = \frac{3}{2}x - 3$
- ✓ $\frac{y-3}{x-4} = 3$
- ☐ $\frac{y-6}{x-2} = \frac{3}{2}$

PROBLEM 2

A line with slope $\frac{3}{2}$ passes through the point $(1, 3)$.

a. Explain why $(3, 6)$ is on this line.
b. Explain why $(0, 0)$ is not on this line.
c. Is the point $(13, 22)$ on this line? Explain why or why not.

Possible Solutions

a. $(3, 6)$ is on the line because the slope between $(3, 6)$ and $(1, 3)$ is $\frac{3}{2}$.
b. $(0, 0)$ is not on this line because the slope between $(0, 0)$ and $(1, 3)$ is 3 and not $\frac{3}{2}$.
c. $(13, 22)$ is not on the line. The slope between $(13, 22)$ and $(1, 3)$ is $\frac{19}{12}$, which is close to but not equal to $\frac{3}{2}$.

PROBLEM 3

Write an equation of the line that passes through $(1, 3)$ and has a slope of $\frac{5}{4}$.

Possible Solutions

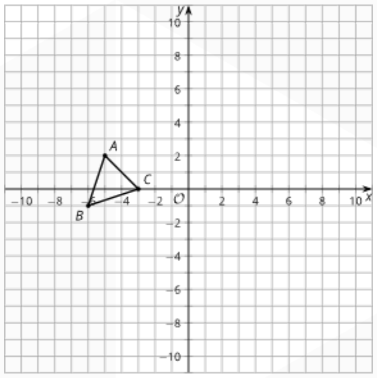
$y = \frac{5}{4}x + \frac{7}{4}$ (or equivalent)

PROBLEM 4

Review Problem

Reflect triangle ABC over the line $x = -6$. Translate the image by the directed line segment from $(0, 0)$ to $(5, -1)$.

What are the coordinates of the vertices in the final image?



Possible Solutions

$A'' = (-2, 1)$, $B'' = (-1, -2)$, and $C'' = (-4, -1)$

REFLECTION AND NOTES