# LESSON 4

# DETERMINING THE PROBABILITY OF THE COMPLEMENT OF AN EVENT

## LEARNING TARGET

- *Given the probability of an event, I can identify and calculate the complement of that event.*

## ALIGNMENT

### Addressing

- **6.DPSR.2.3** Given the probability of an event, identify and calculate the complement of that event.

## LESSON PREPARATION

### Required Materials

- Cool Down

## LESSON NARRATIVE

This is the culminating lesson of the unit building on the foundations of probability of simple events. Through exploration and collaboration, students develop a conceptual understanding of complements of events and then apply that awareness to determine the probability of the complement of an event (MPS.C.1).

## WARM-UP | WHAT SONG? *5–8 minutes*

### Instructional Routines

- MLR6: Three Reads
- Think-Pair-Share

Students review how to calculate the probability of a given event and use this knowledge later in the lesson when they explore the concept of complementary events.

## LAUNCH

Arrange students in groups of 2. Tell students that a music genre is a category of music that describes the form, style, and cultural influence of the music (song) being classified. Consider allowing 1–2 minutes (min.) to use the Three Reads instructional routine to make sure students understand what they are being asked to do. Provide students 1–2 min. of quiet work time to answer the questions, and then ask them to share their thinking with their partners and work together to resolve differences in their answers. Monitor students' work and discussions to identify thinking to highlight during the activity debrief and to identify any students who may need extra support for the following activity.

### Support for English Language Learners

*Reading: MLR6 Three Reads*. Use this routine to support reading comprehension of this activity. Use the first read to orient students to the situation. Ask students to read only the introductory prompt and describe what the situation is about without using numbers (songs of specific genres being randomly shuffled in a playlist). Use the second read to identify quantities and relationships. Ask students what can be counted or measured without focusing on the values (total songs in the playlist, number of songs per genre). After the third read, ask students to brainstorm what they need to know to answer the questions that follow the introduction: "How many songs can be first in a playlist?", "How many songs are from the hip-hop genre?", or "How many songs are not from the pop genre?" This helps students connect the language in the word problem and the reasoning needed to solve the problem while maintaining the cognitive demand of the task.

*Design Principle: Support sense-making*

## STUDENT ACTIVITY

There are $50$ songs in a playlist set to play on shuffle, so the order of the songs played will be random. Genre-wise, $20$ of the songs are pop, $15$ songs are hip-hop, $10$ songs are rock, and $5$ are country.

1. What is the probability that the first song played will be the first song in the playlist?
2. What is the probability that the genre of the first song played will be hip-hop?
3. What is the probability that the first song played won't be a pop song?

## Possible Responses

1. $\frac{1}{50}$ or equivalent
2. $\frac{3}{10}$ or equivalent
3. $\frac{3}{5}$ or equivalent

## Activity Synthesis

Invite previously identified students to share their responses to each question, allowing other students to check their understanding. Ask whether any students have a different answer (it is likely that students will have different but equivalent answers). Reiterate that equivalent answers are acceptable.

# EXPLORATION ACTIVITY | OTHER OUTCOMES *12–15 minutes*

## Instructional Routines

- MLR5: Co-Craft Questions
- Notice and Wonder

Students explore and make sense of what the complement of an event is in a random experiment. By engaging with their groups as they work through the activity, students develop their understanding of a complement of an event (MPS.RC.1).

## LAUNCH

Arrange students in groups of 3–4. Direct students to the image of the spinner and the table of marble color distribution without having them read the accompanying problems. Ask students to take 1 min. to see what they notice and wonder about the displays. Then, provide an additional min. to share with their groups the things they notice and wonder. Encourage groups to ask clarifying questions to deepen their understanding of the displays before they continue with the activity (MPS.RC.1). Listen to conversations, and steer students to wonder something mathematical about the displays if necessary.

Then, give students 7–8 min. of collaborative work time to complete the problems. Consider providing more guidance in a small-group setting for students whom you identified as struggling during the Warm-Up. Follow up with a whole-class discussion.

### Support for English Language Learners

*Speaking, Reading: MLR5 Co-Craft Questions.* Use this routine to help students interpret the language of probabilities. Invite groups to create 1 or 2 mathematical questions that could be answered using the spinner or the table showing the marble color distribution. Note questions that vary in complexity, making sure to share examples that ask about probability of a single event, a combination of events, or of an event not occurring, if possible. Allow students to ask peers clarifying questions regarding how the displays can be used to answer the questions if needed. This will help students make sense of the language of probabilities prior to deepening their understanding of a complement of an event.

*Design Principle(s): Maximize meta-awareness; Support sense-making*

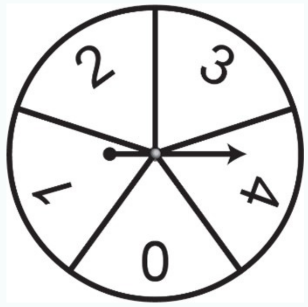
### Support for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "First, I listed the outcomes ______ because . . . ," "I calculate $P$(______) by . . . ," or "How did you get . . . ?"

*Supports accessibility for: Language; Social-emotional skills*

## STUDENT ACTIVITY

A game spinner with 5 equal sections is shown.




1. List all of the possible outcomes.
2. Event A is spinning an even number.
   a. List all outcomes in event A.
   b. $P(\text{A}) =$ ______________

The *complement* of event A is all of the other outcomes not in event A. The complement of event A is notated as "not A."

   c. List all outcomes not in event A.
   d. $P(\text{not A}) =$ ______________

A bag contains 10 colored marbles. The color distribution is shown in the table.

| Green | Blue | Purple | Pink |
|---|---|---|---|
| 4 | 3 | 2 | 1 |

3. Event M is randomly drawing a pink or green marble from the bag.
   a. $P(\text{M}) =$ ______________
   b. List all outcomes not in event M.
   c. $P(\text{not M}) =$ ______________
4. What is the sum of $P(\text{A})$ and $P(\text{not A})$?
5. What is the sum of $P(\text{M})$ and $P(\text{not M})$?
6. Using your answers from questions 5 and 6, what can you conclude about the probabilities of an event and its complement?

### Possible Responses

1. 0, 1, 2, 3, 4
2.
   a. 0, 2, 4
   b. $\frac{3}{5}$ or equivalent
   c. 1, 3
   d. $\frac{2}{5}$ or equivalent

3.
   a. $\frac{1}{2}$ or equivalent
   b. green and purple
   c. $\frac{1}{2}$ or equivalent

4. $1$
5. $1$
6. Answers will vary. The sum of the probabilities of an event and its complement is $1$.

## Anticipated Misconceptions

Students may initially disagree with answers offered in different but equivalent forms. Remind them that equivalent fractions, decimals, and percentages are all acceptable ways to express the probability of events and their complements.

## Activity Synthesis

This discussion ensures students understand how to determine the probability of an event and its complement and realize that the sum of an event and its complement is always 1. Consider using the following questions to facilitate the discussion:

- "What is a complement of an event?" (All the undesired outcomes that can occur in a random experiment)
- "How do you find the probability of a complement of an event?" (Determine the outcomes not in the event, and express this as a probability or subtract the probability of the event [desired outcomes] from 1.)
- "Using the spinner, let event T be the probability of spinning a 2 or 3. What is $P(\text{T})$? What is $P(\text{not T})$?" ($\frac{2}{5}$ or equivalent; $\frac{3}{5}$ or equivalent)
- "Corine determined the sum of the probability of an event and its complement is 90%. Is this possible?" (No. An event and its complement together include all possible outcomes in a random experiment, so the sum of their probabilities will always be $100\%$.)

Refer students to the Student Lesson Summary, which summarizes these key takeaways for reference, before moving on to the next activity. Students can refer to this as needed when practicing collaboratively.

# COLLABORATIVE ACTIVITY | FINDING COMPLEMENTS OF EVENTS *10–12 minutes*

## Instructional Routines

- MLR3: Clarify, Critique, Correct
- Poll the Class

This activity deepens students' skills at finding the complement of an event by prompting them to explain their thinking with mathematical statements or in writing. It also provides students with an opportunity to analyze and correct flawed thinking through an error analysis problem (MPS.AJ.1).

## LAUNCH

Students continue working in their groups. Give students 0.5–1 min. of quiet work time for problem 1, parts a–b. Then for part 1c, before they do any writing, ask students to quietly think about the question for 0.5–1 min. before a brief pause to share their thinking with their groups. Encourage partners to ask clarifying questions to deepen their understanding and to be prepared to justify their thinking for 1c with mathematical statements or in writing. Prompt students to continue with the activity. Consider pausing students' work again after problem 5 and directing students to use a routine similar to problem 1c for problem 6. Wrap up with a whole-group discussion after problem 6.

### Support for English Language Learners

*Writing, Listening, Conversing: MLR3 Clarify, Critique, and Correct.* Use this routine to help students clarify a common error when converting between decimals and percentages. Invite students to work with their groups to identify the error in Ronan's thinking and discuss a correct response before putting anything in writing. Look for and amplify mathematical use of language involving complements of events and processes for correctly converting between decimals and percentages. This will help students affirm their understanding of common errors and how to avoid them and correctly express probabilities in different but equivalent forms.

*Design Principle(s): Optimize output (for explanation); Cultivate conversation; Maximize meta-awareness*

### Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. Check in with students after problems 1, 2, and 5, before a whole-class discussion after problem 6. If necessary, remind students how to determine probabilities of events and their complements.

*Supports accessibility for: Organization; Attention*

## STUDENT ACTIVITY

1. Event A is randomly choosing a letter A from the word *QUADRANT*.
   a. Find $P(\text{A})$.
   b. Find $P(\text{not A})$. Show or explain your thinking.
   c. What is another way to determine $P(\text{not A})$ in addition to the method used to answer the previous question?

2. The table shows the distribution of chocolate candies in a candy jar.

| Milk | Dark | White |
|---|---|---|
| 7 | 3 | 15 |

   a. Find $P(\text{milk})$.
   b. Find $P(\text{not milk})$.
3. If $P(\text{C}) = 25\%$, determine $P(\text{not C})$.
4. If $P(\text{not D}) = \frac{14}{25}$, determine $P(\text{D})$.
5. If $1 - P(\text{B}) = 0.99$, determine $P(\text{B})$.
6. When asked to find the complement of $P(\text{E}) = 0.06$ as a percentage, Ronan incorrectly stated $P(\text{not E}) = 40\%$. Explain the error in Ronan's thinking.

## Possible Responses

1.
    a. $\frac{1}{4}$ or equivalent
    b. $\frac{3}{4}$ or equivalent; sample work: $1 - \frac{1}{4} = \frac{3}{4}$
2. Answers will vary but should be a different method than used in 1b. Sample responses: Subtract $P(\text{A})$ from $1$. Determine the outcomes not in event $A$, and express the result as a probability.
3.
    a. $\frac{7}{25}$ or equivalent
    b. $\frac{18}{25}$ or equivalent
4. $75\%$
5. $\frac{11}{25}$
6. $0.01$
7. Answers will vary. Ronan mistakenly interpreted the value $0.06$ to be $60\%$ when determining $P(\text{not E})$.

## Activity Synthesis

Focus the discussion on students' responses to problems 1c and 6. If not detailed in students' explanations, highlight that when the probability of an event is known, it is more efficient to find the probability of its complement by subtracting that value from 1. Similarly, if the probability of the complement of an event is known, the probability of the event can be found by subtracting that value from 1. Refer to the second paragraph in the Student Lesson Summary to reiterate this concept.

If students seem to have a good grasp of the skills explored in this activity, consider enriching the discussion with additional questions if time allows. Example questions are shown. Use Poll the Class after each question to informally assess student understanding.

- "Event G is the probability of randomly choosing the letter G from the word *SWIMMING*. What is $P(\text{not G})$?" $\left(\frac{7}{8}\right)$
- "If $1 - P(\text{not H}) = 97\%$, what is $P(\text{not H})$ expressed as a decimal?" $(0.03)$
- "Write a mathematical statement that can be used to find $P(\text{B})$ if $P(\text{not B}) = \frac{7}{10}$." $(1 - P(\text{B}) = \frac{7}{10}$ or $1 - \frac{7}{10} = P(\text{B}))$

## LESSON SYNTHESIS

Students learned about a complement to a random event and how to determine the probability of an event's complement. Wrap up by using the Student Lesson Summary to reiterate the main takeaways from the lesson before allowing students to practice the skills independently.

## Student Lesson Summary

Recall that an *event* is a desired outcome in a random experiment. The *complement* of an event is all of the other (undesired) outcomes that can occur. In some random experiments, like spinning a spinner with 5 sections or rolling a number cube, there may be multiple possible outcomes, but when exploring complements, the events are defined as the desired and undesired outcomes only.

The sum of the probabilities of an event and its complement is always $1$. Therefore, the complement of an event, A, can be found using the equation $1 - P(\text{A}) = P(\text{not A})$. It is also true that $1 - P(\text{not A}) = P(\text{A})$ and $P(\text{A}) + P(\text{not A}) = 1$.

## COOL DOWN | SHOW ME THE MONEY! *5 minutes*

### STUDENT ACTIVITY

Pedro has a jar containing 100 coins. The table shows the distribution of the types of coins in the jar.

| Pennies | Nickels | Dimes | Quarters |
|---|---|---|---|
| 48 | 21 | 16 | 15 |

Pedro and his brothers are taking turns reaching into the jar without looking.

1. Event A is getting a dime on the first try. Find $P(\text{not A})$.
2. Event B is getting a nickel on the first try. Find $P(\text{not B})$.
3. The possible outcomes for the complement of event C are penny, nickel, and dime. What is event C?

### Possible Responses

1. $\frac{21}{25}$ or equivalent
2. $\frac{79}{100}$ or equivalent
3. Answers will vary. Event C is selecting a quarter on the first try.

## PRACTICE PROBLEMS

### PROBLEM 1

A playlist contains $25$ songs from a variety of artists. The artist distribution is shown in the table.

| Pink Floyd | Bob Marley | Elvis | The Beatles |
|---|---|---|---|
| 3 | 9 | 5 | 8 |

Event $A$ is randomly playing a Beatles song first. Find $P(\text{not A})$.

#### Possible Solutions

$\frac{17}{25}$

### PROBLEM 2

Event B is randomly choosing a letter E from the word *PRETZELS*.

a. List all outcomes of event B.
b. Find $P(\text{B})$.
c. Find $P(\text{not B})$. Show or explain your thinking.

### Possible Solution

**a.** E

**b.** $\frac{1}{4}$ or equivalent

**c.** $\frac{3}{4}$ or equivalent. Sample explanation: $1 - P(\text{B}) = P(\text{not B})$

## PROBLEM 3

The table shows the distribution of cookie flavors in a cookie jar.

**a.** Find $P(\text{sprinkle})$.
**b.** Find $P(\text{not oatmeal})$.

### Possible Solution

**a.** $\frac{2}{5}$ or equivalent

**b.** $\frac{33}{50}$ or equivalent

## PROBLEM 4

*Review Problem*

Select all the expressions that are equivalent to $4b$.

- ☐ $b + b + b + b$
- ☐ $b + 4$
- ☐ $2b + 2b$
- ☐ $b \cdot b \cdot b \cdot b$
- ☐ $b \div \frac{1}{4}$

### Possible Solutions

- ✓ $b + b + b + b$
- ☐ $b + 4$
- ✓ $2b + 2b$
- ☐ $b \cdot b \cdot b \cdot b$
- ✓ $b \div \frac{1}{4}$

## PROBLEM 5

*Review Problem*

Elena is designing a logo in the shape of a parallelogram. She wants the logo to have an area of 12 square inches (sq. in). She draws bases of different lengths and tries to compute the height for each.

**a.** Write an equation Elena can use to find the height, $h$, for each value of the base, $b$.
**b.** Use your equation to find the height of a parallelogram with base 1.5 inches (in.).

### Possible Solutions

**a.** $h = \frac{12}{b}$
**b.** 8 in.

## REFLECTION AND NOTES

# LESSON 3
# FACTORING LINEAR EXPRESSIONS

## Learning Target

- *I can factor linear expressions using a common factor, including the greatest common factor (GCF), and the distributive property.*

## ALIGNMENT

### Addressing

- **7.PAFR.3.2** Identify linear expressions that are equivalent.
- **7.PAFR.3.4** Factor linear expressions with integer coefficients using the greatest common factor (GCF).

## LESSON PREPARATION

### Required Materials

- Cool Down

## LESSON NARRATIVE

This lesson activates prior knowledge of common factors and greatest common factors from prior grades to use when factoring expressions. Using area models, students explore factoring expressions that have common factors. The area model, familiar from elementary grades, connects the concept of using the distributive property to factoring linear expressions that have a common factor.

As the lesson progresses, the scaffolding of the area model is removed to provide additional opportunities for students to notice patterns and structure in repeated calculations (MPS.SP.1). For students who prefer to use the area model, they can and should still do so as needed when moving into more abstract representations. When students explain their reasoning to their peers and during whole-class discussions, encourage them to use contextually appropriate mathematical language to express their ideas (MPS.RC.1).

## WARM-UP | NUMBER TALK: PARENTHESES *5 minutes*

### Instructional Routines

- MLR8: Discussion Supports
- Number Talk

This number talk reminds students that when evaluating expressions, multiplication is done before addition and subtraction. Parentheses are used to indicate when the order should be different. Remembering how this works will be important to recall in this lesson.

## LAUNCH

Display 1 problem at a time. Give students 30 seconds (sec.) of quiet think time for each problem, and ask them to give you a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow up with a whole-class discussion.

### Support for Students with Disabilities

*Memory: Processing Time.* Provide sticky notes or mini whiteboards to aid students with working memory challenges.

### STUDENT ACTIVITY

Find the value of each expression mentally.

1. $2 + 3 \cdot 4$
2. $(2 + 3)(4)$
3. $2 - 3 \cdot 4$
4. $2 - (3 + 4)$

### Possible Responses

1. $14$
2. $24$
3. $-10$
4. $-5$

### Activity Synthesis

The second question is an opportunity to remind students that *next to* implies "multiply." The second expression could be rewritten $(2 + 3) \cdot 4$. Point out that the value of the second expression can also be found using the distributive property: $2 \cdot 4 + 3 \cdot 4$. The fourth expression could also be rewritten $2 - 3 - 4$.

Ask 1–2 students to share their strategies for each problem. Record and display their responses for all students to see. To involve more students in the conversation, consider asking the following:

- "Who can restate ______'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem differently?"
- "Does anyone want to add on to ______'s strategy?"
- "Do you agree or disagree? Why?"

### Support for English Language Learners

*Speaking: MLR 8 Discussion Supports.*

Provide sentence frames to help students explain their strategies. For example, “I noticed that __________, so I __________.” or “First, I __________ because __________.” Allow students the opportunity to share their answers with a partner, prompting them to rehearse what they will say when they share with the full group. Rehearsing provides students with additional opportunities to clarify their thinking.

*Design Principle(s): Optimize output (for explanation)*

## COLLABORATIVE ACTIVITY | EXPLORING EQUIVALENT EXPRESSIONS USING COMMON FACTORS *15–18 minutes*

### Instructional Routines

- Take Turns
- MLR8: Discussion Supports

Students review common factors and how to find the greatest common factor (GCF) of whole numbers. Students then use common factors to rewrite both numeric and algebraic expressions by applying the distributive property.

Students learned about GCF in 5th grade. As students are reviewing this concept, ensure they have a firm understanding of how to find common factors and the GCF. This is foundational for the remainder of the lesson.

## LAUNCH

Arrange students in groups of 2. Give students 1–2 minutes (min.) to read through the review example before starting on problem 1. If needed, pause to review these concepts and answer questions before students continue with the activity.

Allow students 2–3 min. of quiet work time to complete problem 1. Let students discuss with their partner for 1–2 min. to review responses and share their thinking.

Encourage students to use the instructional routine, Take Turns, to complete problem 2. Have students take turns in completing the area model and explaining their thinking (or process) before the next student has a turn. Before moving on to problem 3, select pairs of students to share their responses with the class.

Give students 2–3 min. of quiet work time to complete the activity before beginning a whole-class discussion.

### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.* To support students in explaining their reasoning to their partner, provide sentence frames for students to use, such as: “I know __________ because ... ,” “First, I __________ because ... ,” and “Second, I __________ because ... .”

*Design Principle(s): Support sense-making; Optimize output (for justification)*

## Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* To support the development of organizational skills, check in with students within the first 2–3 min. of work time. Look for students who are correctly rewriting the linear expressions using factors. If students are stuck, invite them to identify all the common factors for each term first. Then, ask students how they could manipulate the expression using the greater common factor between the 2 numbers.

*Supports accessibility for: Memory; Organization*

## STUDENT ACTIVITY

Review with your partner what you learned about common factors in prior grades. Then, work together to complete the problems that follow.

In multiplication expressions, the numbers being multiplied are called *factors*, and the result is called the *product*.

Numbers can have *common factors*. A common factor of 2 numbers is a number that divides evenly into both numbers.

- For example, $1$, $3$, $5$, $9$, $15$, and are factors of $45$, and $1$, $2$, $3$, $4$, $5$, $6$, $10$, $12$, $15$, $20$, $30$, and $60$ are factors of $60$.
- The common factors of $45$ and $60$ are $1$, $3$, $5$, and $15$.

The *greatest common factor (GCF)* is the largest factor that the values share.

- For example, $15$ is the GCF of $45$ and $60$.

Common factors can be used to rewrite numerical expressions.

1. Consider the expression $24 + 32$.
   a. List all the common factors of $24$ and $32$.
   b. Complete the steps to rewrite the sum of $24$ and $32$ using their GCF.
      $24 + 32$
      $(8 \times$ ______ $) + (8 \times$ ______ $)$
      $8($ ______ $+$ ______ $)$
   c. Choose another **common factor** of $24$ and $32$ to write a different expression equivalent to $24 + 32$.
      $24 + 32$
      ( ______ × ______ )+( ______ × _____ )
      ______ ( _____ + _____ )

Common factors can also be used to rewrite equivalent algebraic expressions.

2. Use your understanding of common factors to complete the area models shown. In each area model, the factors are along the dotted lines, and their products are in the rectangular area. The first model has been completed as an example.

| | $5$ | $-2y$ |
|---|---|---|
| $-3$ | $-15$ | $6y$ |

| | $a$ | $-6$ |
|---|---|---|
| $5$ | | |

| | | |
|---|---|---|
| $2$ | $6$ | $-2b$ |

| | | |
|---|---|---|
| | | |
| $-4$ | $-16x$ | $4$ |

| | | |
|---|---|---|
| | | $-x$ |
| | $30$ | $-6x$ |

| | | |
|---|---|---|
| | $2x$ | |
| | $-8x$ | $28$ |

An area model can be used to show the relationship between equivalent forms of algebraic expressions.

In the area models in the previous problem, the factored form of an expression is the (width) × (length) of the larger rectangle. An equivalent expression in expanded form can be represented by the sum of the areas of the smaller rectangles.

**3.** Complete the table using the area models from the previous problem.

| Factored Form | Expanded Form |
|---|---|
| $-3(5-2y)$ | $-15+6y$ |
| $5(a-6)$ | |
| | $6-2b$ |
| | $-16x+4$ |
| | $30-6x$ |
| | $-8x+28$ |

**4.** Compare your work with your partner's. Discuss any differences, and make any corrections, if needed.

## Possible Responses

**1.**

**a.** 1, 2, 4, 8

**b.** $(8\times 3)+(8\times 4)$; $8(3+4)$

**c.** $(4\times 6)+(4\times 8)$; $4(6+8)$

**2.**

| | $5$ | $-2y$ |
|---|---|---|
| $-3$ | $-15$ | $6y$ |

| | $a$ | $-6$ |
|---|---|---|
| $5$ | $5a$ | $-30$ |

| | $3$ | $-b$ |
|---|---|---|
| $2$ | $6$ | $-2b$ |

| | $4x$ | $-1$ |
|---|---|---|
| $-4$ | $-16x$ | $4$ |

| | $5$ | $-x$ |
|---|---|---|
| $6$ | $30$ | $-6x$ |

| | $2x$ | $-7$ |
|---|---|---|
| $-4$ | $-8x$ | $28$ |

**3.**

| Factored Form | Expanded Form |
|---|---|
| $-3(5-2y)$ | $-15+6y$ |
| $5(a-6)$ | $5a-30$ |
| $2(3-b)$ | $6-2b$ |
| $-4(4x-1)$ | $-16x+4$ |
| $6(5-x)$ | $30-6x$ |
| $-4(2x-7)$ | $-8x+28$ |

**4.** No written response required.

### Anticipated Misconceptions

Students may forget to bring the sign with the term or struggle with correctly multiplying and dividing negative numbers. Allow students the opportunity to correct their own thinking as they work through the problems. It is okay to let the mistakes happen, but discuss them during the whole-class discussion.

### Activity Synthesis

Call on a few groups to share their responses for problem 3. Consider calling on students who wrote the factored form by using a common factor instead of the GCF to highlight the difference. Although both are equivalent expressions, the goal is for students to factor using the GCF.

Wrap up the activity with a discussion of the following questions:

- "How can you find the GCF of 2 terms?" (List all the factor pairs for each term, and determine which factors they have in common. The largest factor they share is the GCF.)
- "How can you use the GCF to rewrite an expression given in expanded form into factored form?" (Divide both terms by the GCF. Then, rewrite the expression as the GCF multiplied by the quotient of each term. Check your work by applying the distributive property to ensure the result is the original expression.)
- "How can you rewrite an expression given in factored form into expanded form?" (By using the distributive property to multiply the term outside the parentheses by each term inside the parentheses)
- "How do you know when to factor out a negative sign?" (Generally, if both terms are negative, then the GCF is negative.)

## GUIDED ACTIVITY | APPLYING PROPERTIES OF OPERATIONS TO FACTOR AND EXPAND LINEAR EXPRESSIONS *15–17 minutes*

### Instructional Routines

- MLR7: Compare and Connect

This Guided Activity focuses on using the distributive property to expand and factor an expression. Students use properties of operations to justify equivalence and make decisions about which tools (e.g., the area model) to use (MSP.RC.1). They compare 2 students' work to determine which student(s) is (are) correct (MPS.PS.1). Students observe that expressions can be written in multiple equivalent forms depending on which common factor is used.

## LAUNCH

Arrange students in groups of 2. Guide students through problem 1 using a think-aloud technique to share your thought process and questioning with the class. Allow students 2–3 min. of quiet work time to complete problem 2. Bring the class together for a discussion, and have 2–3 students share their responses. Encourage students to ask clarifying questions and press for more details. With the class, review the examples in the Student Activity following problem 2 that show how the distributive property can be used to expand and factor an expression.

Use your informal observations up to this point in the lesson to determine how your students should engage with guided problems 3 and 4. This can be done using a guided structure, chunking the problem by each part while discussing the solutions and strategies. Optionally, consider having students work collaboratively and discuss the problems with their partners before concluding with a whole-class discussion to share responses.

### Support for English Language Learners

*Representing, Conversing: MLR7, Compare and Connect.* As students share their responses and reasoning with their partner, call students' attention to the different ways their classmates have chosen a common factor for the expression. Wherever possible, amplify student words and actions that involve the language of optimization: factor pairs, greatest common factor, distributive property.

*Design Principle(s): Maximize meta-awareness; Support sense-making*

### Support for Students with Disabilities

*Social-Emotional Functioning: Peer Tutors.* Pair students with their previously identified peer tutors.

*Supports accessibility for: Social-emotional skills*

## STUDENT ACTIVITY

Using an area model to rewrite a linear expression in an equivalent factored form is a method of **factoring an expression**. Factoring is another way of applying the distributive property of multiplication over addition.

| Factored Form | Expanded Form |
|---|---|
| $-4(2w-5)$ | |
| $-(2-3y)$ | |
| | $20x-10$ |
| $4(a-11)$ | |
| | $10-25b$ |
| $-2(3-z)$ | |

1. Without using area models, complete the table so that each row includes equivalent linear expressions.

2. Consider the expression $24x-8$.
   For the expression $24x-8$, Pao and Camiya wrote different factored forms of the expression, as shown.

| Pao's Expression | Camiya's Expression |
|---|---|
| $4(6x-2)$ | $8(3x-1)$ |

   a. Explain who wrote a correct equivalent expression in factored form.
   b. What is another equivalent expression using a different common factor?
   c. Which expression is equivalent to $24x-8$, using the GCF of $24x$ and $-8$?

Properties of operations can be used in different ways to create equivalent expressions.

- The distributive property can be used to expand an expression. For example, the expression $3(x+5)$ can be rewritten as $3(x)+3(5)$ or $3x+15$.
- The distributive property can also be used to factor an expression. For example, the expression $8x+12$ can be rewritten as $2(4x+6)$ or $4(2x+3)$.

Equivalent expressions can also be generated using the commutative property of addition or multiplication as well as the distributive property. An example is shown.

$(4x+6)2 \qquad 4(3+2x)$

3. Complete the following to factor the expression $12d-36$.
   a. Choose a common factor of the terms $12d$ and $-36$. Circle your choice.
      2 3 4 6 12
   b. What is the result of dividing by $12d$ the common factor selected?

c. What is the result of dividing by $-36$ the common factor selected?
d. Use your answers to parts A–C to rewrite an expression equivalent to $12d - 36$ in factored form.

$$12d - 36 = \_\_\_\_\_\_(\_\_\_\_\_\_ + \_\_\_\_\_\_)$$

e. Choose a different common factor from part A, and use it to write a different equivalent expression.

$$12d - 36 = \_\_\_\_\_\_(\_\_\_\_\_\_ + \_\_\_\_\_\_)$$

4. Several expressions are shown.

| | | |
|---|---|---|
| $8(3 + m)$ | $2(8m + 12)$ | $4(2m + 6)$ |
| $(2m + 8)3$ | $(1 + 3)8m$ | $(12 + 4m)2$ |

a. Circle the expressions equivalent to $8m + 24$.
b. Put an asterisk (*) next to the equivalent expression that has the GCF of $8m$ and $24$ as 1 of its factors.
c. Put a smiley face (☺) next to an expression where more than 1 property of operations can be used to show it is equivalent to $8m + 24$.

## Possible Responses

1.

| Factored Form | Expanded Form |
|---|---|
| $-4(2w - 5)$ | $-8w + 20$ |
| $-(2 - 3y)$ | $-2 + 3y$ |
| $10(2x - 1)$ | $20x - 10$ |
| $4(a - 11)$ | $4a - 44$ |
| $5(2 - 5b)$ | $10 - 25b$ |
| $-2(3 - z)$ | $-6 + 2z$ |

2. Answers will vary. Sample response: Both Pao and Camiya have correct expressions, but Camiya's uses the GCF, and Pao uses a smaller common factor.
   a. $2(12x - 4)$
   b. Camiya's expression, $8(3x - 1)$
3. Answers will vary. Sample responses:
   a. $12$
   b. $d$
   c. $-3$
   d. $12d - 36 = 12(d + -3)$
   e. Answers will vary. Sample response: $12d - 36 = 6(2d + -6)$
4. a. $8(3 + m)$; $4(2m + 6)$; $(12 + 4m)2$
   b. $8(3 + m)$
   c. $8(3 + m)$; $(12 + 4m)2$

## Activity Synthesis

Call on 3–4 students to share their responses to problems 3 and 4. To highlight different responses, select students who picked different common factors for problem 3. Engage the class in a discussion using the following questions:

- "How did you determine which factor to use in problem 3?"
- "How did you determine which expressions in problem 4 were equivalent to $8m + 24$?"
- "Can you identify the properties of operations that can be used to show that the expressions in problem c are equivalent to $8m + 24$?"

- "How can the distributive property be used to expand an expression?" (By distributing the coefficient of the parenthesis to each term inside the parenthesis)
- "How can the distributive property be used to factor an expression?" (By determining the common factor and factoring out that value from each term)

Note that although multiple equivalent expressions are possible, the goal is to factor an expression using the GCF.

## LESSON SYNTHESIS

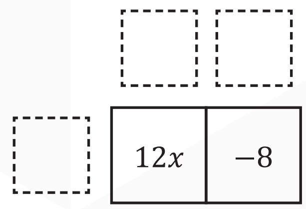
The goal of this lesson is for students to be able to rewrite linear expressions in either expanded form or factored form using the distributive property. Review the example in the Student Lesson Summary that highlights how the distributive property is used in each step.

### Student Lesson Summary

Properties of operations can be used in different ways to generate equivalent expressions. The distributive property can be used to expand an expression. For example, $3(x+5)=3x+15$. The distributive property can also be used in the other direction to factor an expression. For example, $8x+12=4(2x+3)$.

The work of using the distributive property to factor the expression $12x-8$ can be represented by an area model.

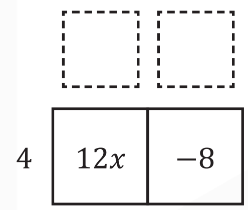
- The terms of the product go inside the area of the rectangle, as shown.
- Use the expression to find the *greatest common factor* of both terms.

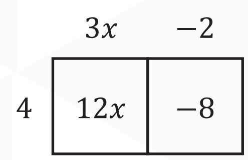



> The **greatest common factor** (GCF) of 2 or more whole numbers is the largest whole number that evenly divides the given whole numbers.

The terms $12x$ and $-8$ each have a GCF of $4$. The GCF is placed on one side of the large rectangle to represent one dimension of the rectangle, as shown.

- To determine the other dimension of the rectangle using the area model, think "$4$ times *what* is $12x$?" and "$4$ times *what* is $-8$?" and write the other factors along the top dimension of the rectangle, as shown.




The factors resulting in a product of $12x-8$ are $4$ and $(3x-2)$.
So, $12x-8$ is equivalent to $4(3x-2)$.

## COOL DOWN | FACTORING A LINEAR EXPRESSION *5 minutes*

### STUDENT ACTIVITY

Janelle missed math class. She understands using the distributive property to expand an expression, but she doesn't know how to rewrite the sum of 2 terms with a common factor in factored form.

Explain to Janelle how to rewrite an expression like $36a-16$ in an equivalent factored form.

### Possible Responses

Answers will vary. First, find a common factor of the 2 terms, $36a$ and $-16$, such as $4$. Divide each term by the common factor to get $9a-4$. The equivalent factored form is the product of the 2 factors, $4(9a-4)$.

## PRACTICE PROBLEMS

### PROBLEM 1

Fill in the blanks of the equation to make it true.

$$75a+25b= \_\_\_\_\_\_ (\_\_\_\_\_\_a+b)$$

**Possible Solutions**

$75a+25b=\underline{\ \ 25\ \ }(\underline{\ \ 3\ \ }a+b)$

### PROBLEM 2

Rewrite each expression in an equivalent factored form.

**a.** $-15z+20=$ ____________________

**b.** $4x-32=$ ____________________

**c.** $-27-12d=$ ____________________

**d.** $35+28g=$ ____________________

**Possible Solutions**

**a.** $-5(3z-4)$

**b.** $4(x-8)$

**c.** $-3(9+4d)$

**d.** $7(5+4g)$

### PROBLEM 3

**a.** Expand to write an equivalent expression: $\frac{-1}{4}(-8x+12y)$

**b.** Factor to write an equivalent expression: $36a-16$

**Possible Solutions**

**a.** $2x-3y$

**b.** $4(9a-4)$ or $2(18a-8)$

### PROBLEM 4

*Review Problem*

Elena makes her favorite shade of purple paint by mixing 3 cups (c.) of blue paint, $1\frac{1}{2}$ c. of red paint, and $\frac{1}{2}$ of a c. of white paint. Elena has $\frac{2}{3}$ of a c. of white paint.

**a.** Assuming she has enough red paint and blue paint, how much purple paint can Elena make?

**b.** How much blue paint and red paint will Elena need to use with the $\frac{2}{3}$ of a c. of white paint?

**Possible Solutions**

**a.** $\frac{20}{3}$ c.

**b.** $4$ c. of blue paint; $2$ c. of red paint

# LESSON 3
# DOMAIN AND RANGE

## LEARNING TARGETS

- *I can identify the domain and range of relation or function using inequalities.*
- *I can identify the domain and range of a relation or function using a list.*

## ALIGNMENT

### Addressing

- **8.PAFR.1.4** Describe the key features of given functions, including *domain*, *range*, *intervals* of *increasing* or *decreasing*, *constant*, *discrete*, *continuous*, and *intercepts*.

## LESSON PREPARATION

### Required Materials

- Cool Down

## LESSON NARRATIVE

In prior lessons, students developed an understanding of inputs, outputs, and different representations of relations. In this lesson, students are introduced to *domain* and *range* as sets of input and output values and analyze multiple representations of relations and functions to determine the best way to identify domain and range either as a list of numbers or as an inequality (MPS.AJ.1). Students are also introduced to *compound inequality* notation to represent domain and range.

This lesson establishes the foundation for students exploring other key features of relations and functions such as intervals over which a function is increasing, decreasing, constant, discrete, or continuous. Each are represented as intervals of the domain in the same way that the domain and range are for an entire function. These concepts will be explored in upcoming lessons.

## WARM-UP | CREATE A PATTERN *5 minutes*

In this Warm-Up, students develop a pattern of sequential numbers using a list of possible digits, prompting them to think flexibly about how numbers increase or decrease by the same amount in a pattern (MPS.SP.1).

### LAUNCH

Give students 1–2 minutes (min.) of quiet work time to complete the task. If some students complete the task quickly, encourage them to try to come up with another pattern. Invite 4–5 students to share their patterns with the class during the whole-class discussion.

### STUDENT ACTIVITY

1. Place a digit in each blue box to create a pattern where the next number changes by the same amount each time. Use only the digits $0$ to $9$, without repeating any digits.

☐☐, ☐☐, ☐☐, ☐☐ . . .

#### Possible Responses

Answers will vary. Sample responses: $19, 28, 37, 46$

#### Activity Synthesis

Invite students to share their patterns and their justification(s) of why their pattern works. Highlight explanations that use the vocabulary words *increasing*, *decreasing*, and *term(s)*. This Warm-Up provides students an opportunity to explore a discrete list of values, which is similar to lists they will be using in this lesson.

## GUIDED ACTIVITY | INTRODUCING DOMAIN AND RANGE *15–17 minutes*

#### Instructional Routines

- MLR1: Stronger and Clearer Each Time

This activity introduces several new concepts for students, including domain, range, and compound inequalities. The guidance provided is designed to give students support as they work through different representations of relations and functions in describing their domain and range. Encourage students to use contextually appropriate vocabulary throughout the activity (MPS.RC.1) to clearly communicate their ideas. As the activity progresses, students are given more opportunities to try the problems without your support. Consider the needs of your students, and remove scaffolds as you deem appropriate.

### LAUNCH

Arrange students in groups of 2–4. Pose problem 1 to all students. Give students 1 min. of quiet think time before inviting volunteers to share their thoughts and ideas. Be prepared to push students' thinking to what makes a mathematical statement true and whether there are many, 1, or no possible values that can be substituted for $x$ and $y$. After finishing problem 1, give students 1 min. to read the paragraph that defines *domain* before modeling problem 2 using a think-aloud strategy. Engage students in a discussion of how to write the domain using inequalities. Consider giving students additional $x$-values such as $100$, $10$, $0.1$, $0.01$ and $-100$,$-10$,$-0.1$,$-0.01$ and determining what the output will be for each. If needed, give students 1 min. to discuss in their groups what they believe the possible $x$-values are in this relation. Invite 2–3 students to share what they predicted for the domain of this relation before explaining the solution to problem 2d.

Explain to students that just as the domain identifies all the possible values of the input (or $x$) of a relation or function, the range identifies all the possible values of the output (or $y$) of the relation or function. Use a structure similar to problem 2 for problem 3, unless students need a different level of scaffolding based on their interactions with the previous problem.

Give students 3–4 min. to complete problems 4 and 5 in their groups before a whole-class discussion. Identify students who wrote domain and range using a list instead of using inequalities in problems 4 and 5. Use these student responses to facilitate the whole-class discussion.

### Support for English Language Learners

*Speaking, Listening: MLR1 Stronger and Clearer Each Time.* To help students develop a strong understanding of the terms *domain* and *range*, ask students to write a response to the question "How do you know if the list of values represents the domain or the range of a relation?" Use successive pair shares to give students a structured opportunity to revise and review their response. Provide prompts for feedback that will help students strengthen their ideas and clarify their language (e.g., "Can you explain how . . . ," "You should expand on . . .").

*Design Principle(s): Support sense-making; Optimize output (for explanation)*

### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Represent the same information through different modalities by using rule diagrams, tables, mappings, and graphs. To support students in thinking of appropriate domain and range for a given relation, suggest they create a representation that is the most useful for them to interpret and visualize.

*Supports accessibility for: Conceptual processing; Visual-spatial processing*

### STUDENT ACTIVITY

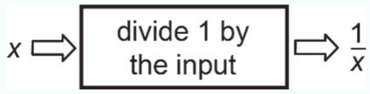
1. Consider the statements shown.
   $12 \div 3 = 4$ because $12 = 4 \cdot 3$.
   $4 \div x = 1$ because $4 = x \cdot 1$.
   $6 \div 0 = y$ because $6 = y \cdot 0$·
   **a.** In the second statement, what value can be used in place of $x$ so that the statement is true?
   **b.** In the third statement, what value can be used in place of $y$ so that the statement is true?

The inputs of a relation are described as its *domain*. The domain of a relation can be written as a list of values or as an inequality. In some cases, 1 or more values are not allowed in the domain of a relation. The domain for a relation can be represented using inequalities that include all possible input values defined as $x$.

**2.** Consider the rule shown for a relation.

**a.** Complete the table of values using the relation.




| $x$ | $-\frac{3}{7}$ | 1 | $\frac{4}{5}$ | 0 | 12 |
|---|---|---|---|---|---|
| $y$ | $-\frac{7}{3}$ | | | | |

**b.** Identify any values for $x$ that are not allowed in the domain of the relation?

**c.** Work with your partner to determine if there is another input that is not allowed for the relation $\frac{1}{x}$.

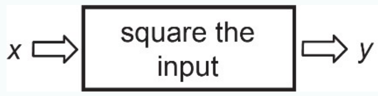
**d.** Complete the statements to identify the domain of the relation.

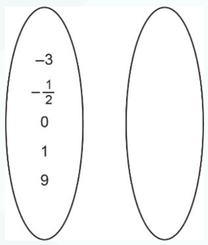
The domain of the relation is $x >$ ______ or $x <$ ______. These 2 inequalities can also be written as one statement, $x \neq$ ______.

The outputs of a relation are described as its *range*. The range of a relation can be written as a list of values or as an inequality. In some cases, 1 or more values are not possible in the range of a relation. The range of a relation can be represented using inequalities that include all possible output values defined as $y$.

**3.** Consider the rule shown for a relation.

**a.** Complete the mapping diagram for the relation.



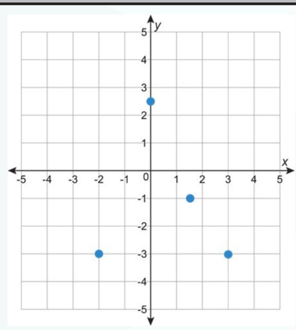
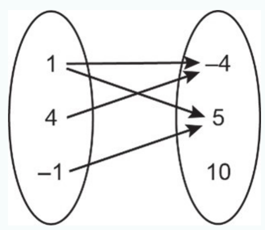

**b.** Based on the rule, what type of output values are not possible for this relation?

**c.** Write an inequality to represent the range of this relation.

$y$ ◯ ____

In some cases, the domain and range cannot be written as an inequality because the relation includes only specific values. In such cases, the domain and range are written as a set of numbers within brackets, { and }

**4.** For each relation, write the domain and range as a set of numbers.

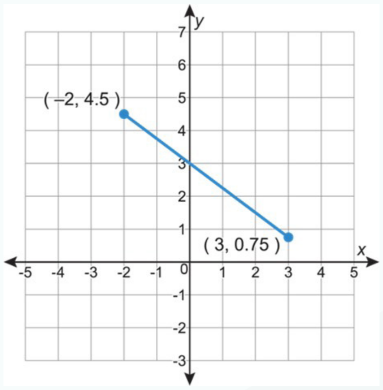
| Relation A | Relation B |
|---|---|
|  Domain: ______________ Range: ______________ |  Domain: ______________ Range: ______________ |



There are also instances where the domain and range of a relation can be represented by a *compound inequality*. For example, $-4 \le x \le \frac{3}{5}$ is a compound inequality that reads as "$-4$ is less than or equal to $x$, which is less than or equal to $\frac{3}{5}$." This inequality is true for any value of $x$ between $-4$ and $\frac{3}{5}$, including either of those values.

**5.** A graph is shown.




**a.** Represent the domain and range of the graph using compound inequalities.
Domain: ________________________
Range: ________________________

**b.** Compare this graph to relation A in the previous problem. Discuss with your partner why the domain and range of this graph could not be represented by writing the elements of each in a list.

## Possible Responses

**1.**

**a.** When $x=4$, the statement will be true

**b.** Answers will vary. Sample response: There is no number that can be multiplied by $0$ to result in $6$, so there is no value for $x$ that can be substituted into the statement to make it true.

**2.**

**a.**

| $x$ | $-\frac{3}{7}$ | $1$ | $\frac{4}{5}$ | $0$ | $12$ |
|---|---|---|---|---|---|
| $y$ | $-\frac{7}{3}$ | $1$ | $\frac{5}{4}$ | Does not exist | $\frac{1}{12}$ |

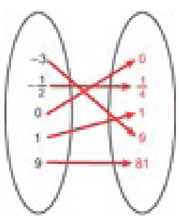
**b.** $0$

**c.** There are no other inputs that are not allowed.

**d.** $0; 0; 0$

**3.**

**a.**



**b.** Negative values are not possible.

**c.** $y \ge 0$

4. Relation A: Domain $\{-2, 0, 1.5, 3\}$; Range $\{-3, -1, 2.5\}$
   Relation B: Domain $\{-1, 1, 4\}$; Range $\{-4, 5\}$

5.
   a. Domain: $-2 \leq x \leq 3$
      Range: $0.75 \leq y \leq 4.5$
   b. No written response expected. Sample discussion: The graph only has points on the interval described; all other $x$- and $y$-values do not have a point on the graph.

### Activity Synthesis

Invite selected students to share their responses for problems 4 and 5. Use the presented solutions to ask students the following guiding questions:

- "When are domain and range identified as a list?" (When the relation is discrete)
- "When are domain and range identified as an inequality?" (When the relation is continuous)
- "Can you come up with an example of a relation in which the domain is a list and the range is an inequality?" (A vertical line would create such a relation.)

## COLLABORATIVE ACTIVITY | IDENTIFYING DOMAIN AND RANGE OF A RELATION *10–13 minutes*

### Instructional Routines

- MLR7: Compare and Connect
- Poll the Class

This activity asks students to identify the domain and range of relations and functions that are presented using different representations and engages students in making decisions about how to represent the key features using a list or an inequality.

## LAUNCH

Assign students to groups of 2–3. Give students 5–7 min. to work together to complete the activity. Monitor student discussion during the activity to identify students that may need additional support. Consider asking the following questions to groups as they work if students need a starting point to determine the domain and range of each representation.

- "Can you name 1 $x$-value that is part of the relation? 1 $y$-value?"
- "Can you identify all the possible $x$-values in a list? Or an inequality?"
- "Can you identify all the possible $y$-values in a list? Or an inequality?"

After collaborative work, invite students to share their descriptions of the domain and range for each relation during a whole-class discussion.

### Support for English Language Learners

*Representing, Speaking: MLR7 Compare and Connect.* Use this routine to give students an opportunity to compare approaches to finding domain and range of different relations. Ask students to share their approach with their group. Invite groups to discuss what is the same and what is different about their strategies for determining the domain and range. Listen for opportunities to highlight language such as *inequality*, *list*, *input*, *output*, and *possible values*.

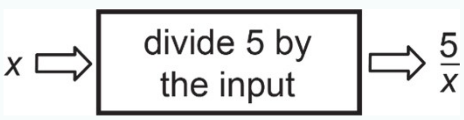
*Design Principle(s): Optimize output; Cultivate conversation*

## Support for Students with Disabilities

*Representation: Internalize Comprehension.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. For example, consider only asking students to work on the domain of the first 2 relations and then on the range. Pause between each problem and key feature to ask 1–2 students to explain their domain/range before revealing the correct answer.

## STUDENT ACTIVITY

1. Identify the domain and range of each relation. Represent the domain and range as lists or using inequalities.

<table>
<tr><th>Relation</th><th>Domain</th></tr>
<tr><td rowspan="3">
<table>
<tr><th>$x$</th><th>$y$</th></tr>
<tr><td>−4.7</td><td>7</td></tr>
<tr><td>2.4</td><td>4.7</td></tr>
<tr><td>−1.6</td><td>3.9</td></tr>
<tr><td>8</td><td>10</td></tr>
</table>
</td><td></td></tr>
<tr><th>Range</th></tr>
<tr><td></td></tr>
<tr><th>Relation</th><th>Domain</th></tr>
<tr><td rowspan="3">{(2.3,7.4), (3.8,6), (1.9,5.2), (2.3,8)}</td><td></td></tr>
<tr><th>Range</th></tr>
<tr><td></td></tr>
<tr><th>Relation</th><th>Domain</th></tr>
<tr><td rowspan="3">
</td><td></td></tr>
<tr><th>Range</th></tr>
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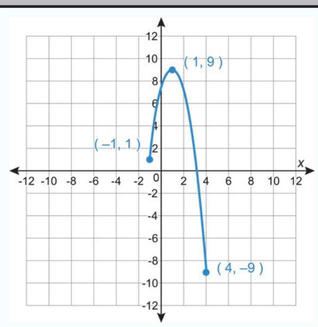
<tr><td rowspan="3">
</td><td></td></tr>
<tr><th>Range</th></tr>
<tr><td></td></tr>
</table>

### Possible Responses

Table of Values: Domain $\{-4.7, -1.6, 2.4, 8\}$, Range $\{3.9, 4.7, 7, 10\}$

List: Domain $\{1.9, 2.3, 3.8\}$, Range $\{5.2, 6, 7.4, 8\}$

Rule: Domain: $x < 0$ and $x > 0$, Range: $y < 0$ and $y > 0$

Graph: Domain: $-1 \leq x \leq 4$, Range: $-9 \leq y \leq 9$

### Anticipated Misconceptions

Students may struggle with writing the domain and range of the rule representation of the relation. Consider recommending that students create a table of values for some ordered pairs that satisfy the rule, including a variety of rational numbers as inputs. Ask students, "Is there a possible input that will not produce an output?"

When working with the graph representation, students may incorrectly believe that the range should be between the 2 endpoints of the graph. Use the graph representation to show that the highest value in the range interval is larger than either of the endpoints. Ask students, "Is there a possible output larger (or smaller) than the current range you identified?"

### Activity Synthesis

Facilitate a whole-class discussion by inviting 1–2 students to identify the domain and range for each representation of the relations shown. Consider using Poll the Class to get a sense of which representations may need more discussion or clarification. Pay close attention to the discussion students have when working with the rule representation and graphical representation. Both representations may highlight misconceptions students have regarding range. Consider asking students the following guiding questions:

- "What does a domain of $-2 \leq x \leq 6$ mean?" (The relation is continuous and includes all $x$-values between $-2$ and $6$.)
- "What does a range of $\{-2, -1, 3, 5, 6\}$ mean?" (The relation is discrete with points whose outputs, or $y$-values, are only those included in the list.)

## LESSON SYNTHESIS

In this lesson, students determined the domain and range of different representations of relations and functions. This study of domain and range is foundational because students will continue to apply this knowledge in upcoming lessons focusing on other important features of functions. Refer to the Student Lesson Summary to formally review the vocabulary terms introduced in this lesson. Encourage students to create diagrams, examples, nonexamples, and explanations of each of the terms studied in this lesson.

### Student Lesson Summary

The *domain* of a relation can be represented as a list or an inequality of all possible $x$-values that can be used in the relation. The *range* of a relation can be represented as a list or an inequality of all possible $y$-values that can be used in the relation.

**Domain** is the complete set of possible values of the input of a function or relation. The domain may vary depending on the context.

**Range** is the complete set of possible values of the output of a relation or function.

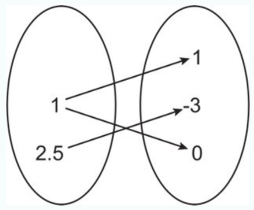
When organized in a list, the values are written in increasing order without duplicates. When represented as an inequality, a *compound inequality* is often used.

A **compound inequality** is a conjunction of two or more inequalities.

## COOL DOWN | MAPPING RELATIONS *5 minutes*

### STUDENT ACTIVITY

1. A relation is shown using a mapping diagram.




   a. Identify the domain of the relation.
   b. Identify the range of the relation

### Possible Responses

a. $\{1, 2.5\}$
b. $\{-3, 0, 1\}$

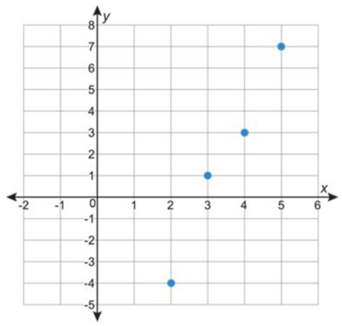
## PRACTICE PROBLEMS

### PROBLEM 1

Five relations are shown. Select **all** of the relations with the domain $\{-4, 1, 3, 7\}$.

☐ $\{(7, 2), (1, 4), (3, 4), (\{-4, \{-1)\}$

☐



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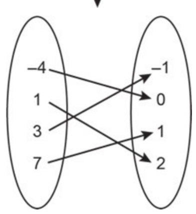
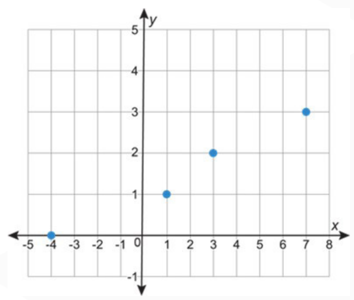



☐

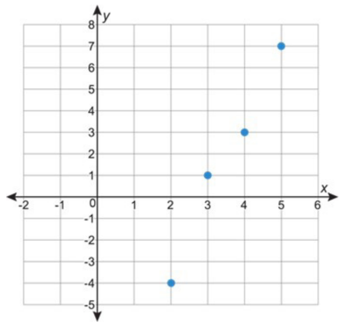
| $x$ | $-4$ | $0$ | $2$ | $6$ |
|---|---|---|---|---|
| $y$ | $-4$ | $3$ | $1$ | $7$ |

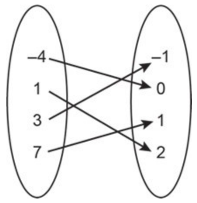
☐



### Possible Solutions

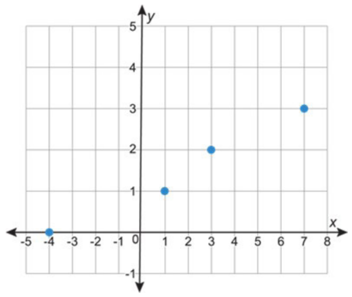
☐ $\{(7, 2), (1, 4), (3, 4), (\{-4, \{-1)\}$

☐ 

☐ 

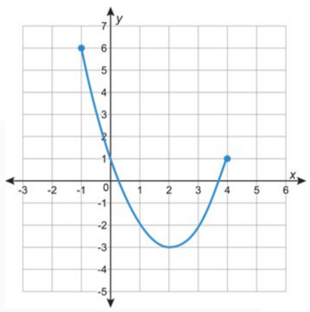
☐

| $x$ | $-4$ | $0$ | $2$ | $6$ |
|---|---|---|---|---|
| $y$ | $-4$ | $3$ | $1$ | $7$ |

☐ 

## PROBLEM 2

The graph of a relation is shown.



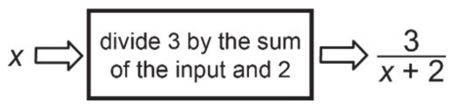
**a.** Identify the domain of the relation.
**b.** Identify the range of the relation.

### Possible Solutions

**a.** $-1 \leq x \leq 4$
**b.** $-3 \leq y \leq 6$

## PROBLEM 3

A rule is shown for a relation.




Identify the domain of the relation.

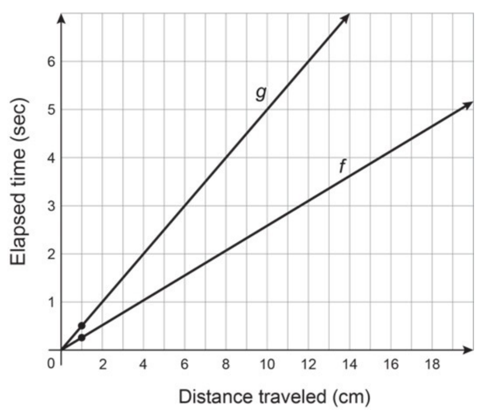
### Possible Solutions

$x < -2$ and $x > -2$

## PROBLEM 4

*Review Problem*

The graphs represent the positions of 2 turtles in a race.




**a.** On the same axes, draw a line for a third turtle that is going half as fast as the turtle represented by line $g$.
**b.** Explain how your line shows that the turtle is going half as fast.

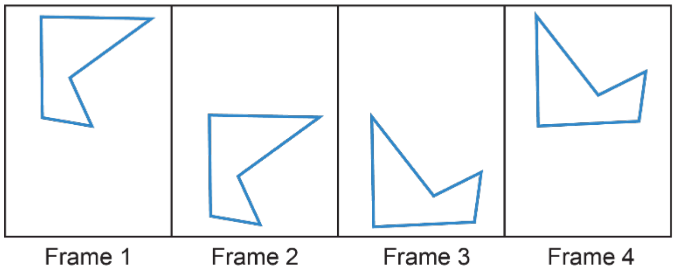
### Possible Solutions

**a.** A line through $(0, 0)$, $(1, 1)$, $(2, 2)$, etc.
**b.** Answers will vary. Sample response: Looking at the values for 2 seconds, turtle $g$ moves 4 centimeters (cm) and the third turtle moves only 2 cm. This third turtle covers half the distance in the same amount of time.

## PROBLEM 5

*Review Problem*

Four successive positions of a shape are shown.




Describe how the shape moves from . . .

- **a.** . . . frame $1$ to frame $2$.
- **b.** . . . frame $2$ to frame $3$.
- **c.** . . . frame $3$ to frame $4$.

### Possible Solutions

- **a.** Slide down.
- **b.** Turn counterclockwise $90°$.
- **c.** Slide down.