Unit 1, Lesson 9: Rearranging Equations of Lines

Warm-Up: Solving Equations

1. Solve each equation by isolating the variable. Show your work.

   a. $14 = 3x - 28$

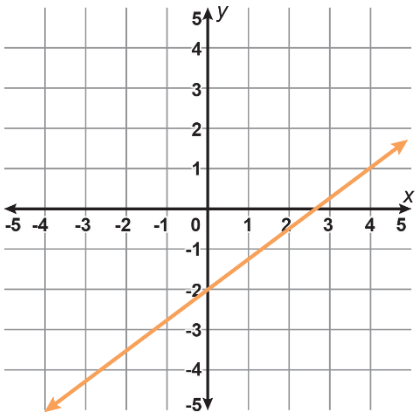
   b. $x + 19 = 6(x - 11)$

   c. $27 + 9y = -12$

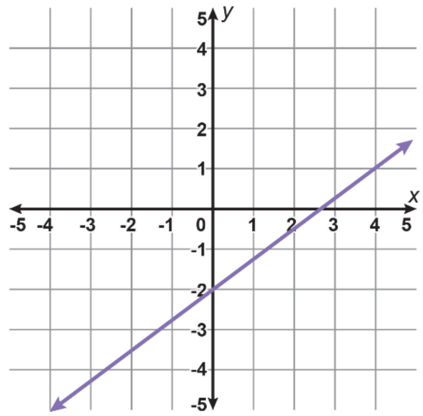
Guided Activity: Different Forms of Linear Equations

Three linear equations and their graphs are shown.

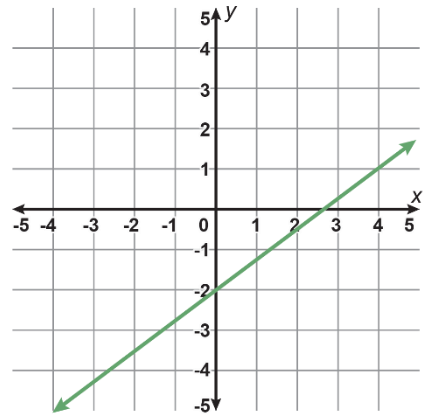
$y = \frac{3}{4}x - 2$



$(y - 1) = \frac{3}{4}(x - 4)$



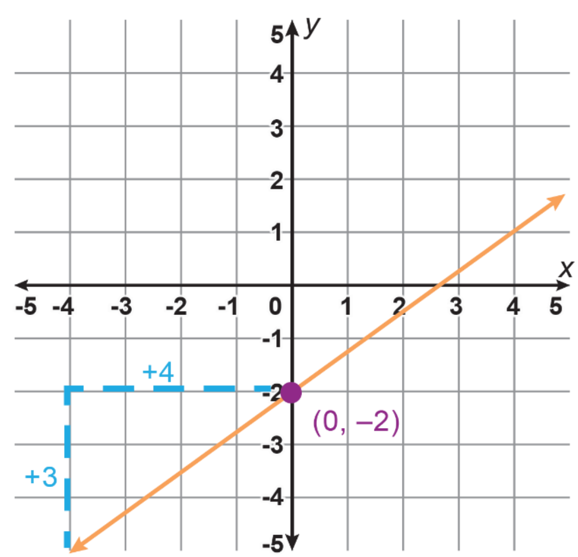
$3x - 4y = 8$



1. Discuss with your partner what you notice about the graphs and equations.

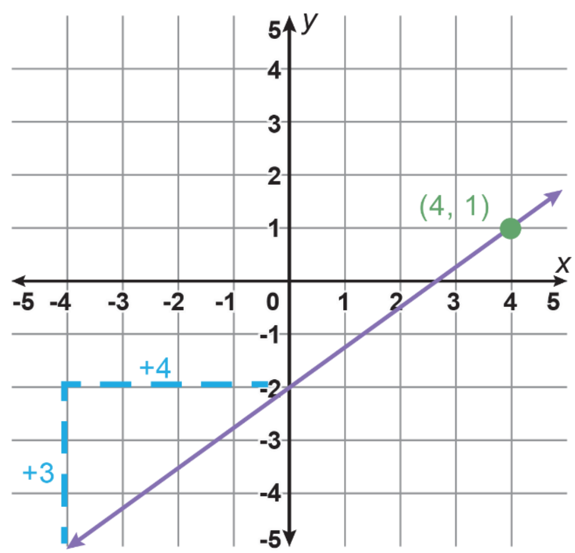
The equations of lines can be written in different but equivalent forms. Each form of the equation relates to different features of the graph, including the slope, y-intercept, and other points on the line.

One form for the equation of a line is *slope-intercept form*.

Slope-Intercept Form	$y = \frac{3}{4}x - 2$
$y = mx + b$ where m is the slope and b is the y-intercept	

2. Discuss with your partner why this form is called slope-intercept form. Summarize your discussion.

A second form for the equation of a line is called *point-slope form*.

Point-Slope Form	$(y-1)=\frac{3}{4}(x-4)$
$(y-y_1)=m(x-x_1)$ where m is the slope and (x_1, y_1) is any point on the line	

Point-slope form can be written using the slope and any point on the line.

3. Compare the equation and the graph of the line. What do you notice about the signs of the coordinates of the green point used to write the equation compared to the values in the equation?

4. Katrina wrote a different equation for the same line in point-slope form using the y-intercept, $(0,-2)$. Her equation is shown.

$$(y+2)=\frac{3}{4}(x-0)$$

a. Explain why the expression $(y+2)$ is on the left side of the equation when the y-value of the point used is -2.

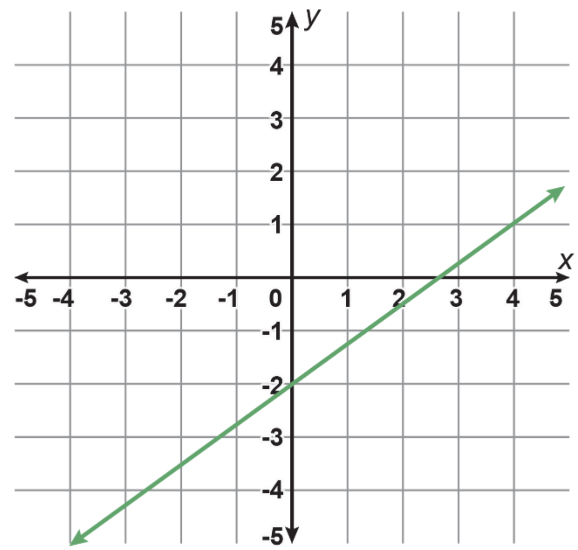
b. Discuss with your partner how the equation $(y+2)=\frac{3}{4}(x-0)$ could be rewritten in slope-intercept form.

c. Rewrite the equation $(y+2)=\frac{3}{4}(x-0)$ in slope-intercept form by isolating the variable y.

d. Rewrite the original point-slope equation, $(y-1)=\frac{3}{4}(x-4)$, in slope-intercept form.

e. What do you notice about the 2 equations you just generated?

The third form for the equation of a line is called *standard form*.

Standard Form	$\mathbf{3x - 4y = 8}$
$\boldsymbol{Ax + By = C}$ where A and B cannot both be zero	

5. Rewrite the equation $3x - 4y = 8$ in slope-intercept form by isolating the variable y.

6. Use the equation $f(x) = \frac{2}{5}x - 4$ to complete the following.

   a. What is the form of the linear equation given?

   b. Rewrite the equation in standard form.

Collaborative Activity: Equivalent Forms

Twelve linear equations are given in various forms.

$4x - 6y = 24$	$-4x - 6y = 24$	$-6x + 4y = 24$	$6x + 4y = 24$
$y = -\frac{2}{3}x - 4$	$f(x) = -\frac{2}{3}x - 6$	$y = \frac{3}{2}x + 6$	$f(x) = \frac{2}{3}x - 4$
$(y + 6) = -\frac{3}{2}(x - 8)$		$(y - 6) = \frac{3}{2}x$	
$(y + 6) = \frac{2}{3}(x + 3)$		$(y - 6) = -\frac{2}{3}(x + 15)$	

1. Using the equations, work with your partner to complete the table so that each row contains a set of equivalent equations, one written in each form. Not all equations will be used.

Equivalent Equations	Standard Form	Slope-Intercept Form	Point-Slope Form
Set 1			
Set 2			
Set 3			

Lesson Summary

Previous courses introduced the *slope-intercept form* for the equation of a line. The slope-intercept form of a line is $y = mx + b$, where m represents the *slope* of the line and b represents the y*-intercept*.

The **slope** of a line is the ratio of the change in the vertical direction to change in the horizontal direction, often expressed as $\frac{\text{change in } y}{\text{change in } x}$, or $\frac{\Delta y}{\Delta x}$.

The $\boldsymbol{y}$**-intercept** is the value of y at the point where a line or graph intersects the y-axis. The value of x is 0 at this point.

In this lesson, 2 additional forms of linear equations were introduced.

- The *point-slope form* of the equation of a line is $(y - y_1) = m(x - x_1)$, where m is the slope and (x_1, y_1) is any point on the line.
- The *standard form* of the equation of a line is $Ax + By = C$, where A and B cannot both be zero.

By using key features revealed in the form of the linear equation given, along with properties of operations and properties of equality, linear equations can be rewritten in other equivalent forms.

Practice Problems

1. Use the equation $(y - 7) = -\frac{1}{3}(x + 2)$ to complete the following.

    a. What is the form of the linear equation given?

    b. Rewrite the equation in standard form.

2. Use the equation $6x - 9y = -12$ to complete the following.

   a. What is the form of the linear equation given?

   b. Rewrite the equation in slope-intercept form.

3. Select **all** of the equations that are equivalent to $(y + 3) = -\frac{1}{4}x$.

- ☐ $y = -\frac{1}{4}x$
- ☐ $y = -\frac{1}{4}x - 3$
- ☐ $(y - 3) = -\frac{1}{4}(x - 1)$
- ☐ $x + 4y = 3$
- ☐ $x + 4y = -12$

4. Is the equation $(y - 5) = \frac{1}{2}(x + 4)$ equivalent to $y = \frac{1}{2}x + 7$? Explain your thinking.

Review Problems

5. A car traveled 180 miles (mi.) at a constant rate.

    a. Complete the table to show the rate at which the car was traveling if it completed the same distance in each number of hours (hr.).

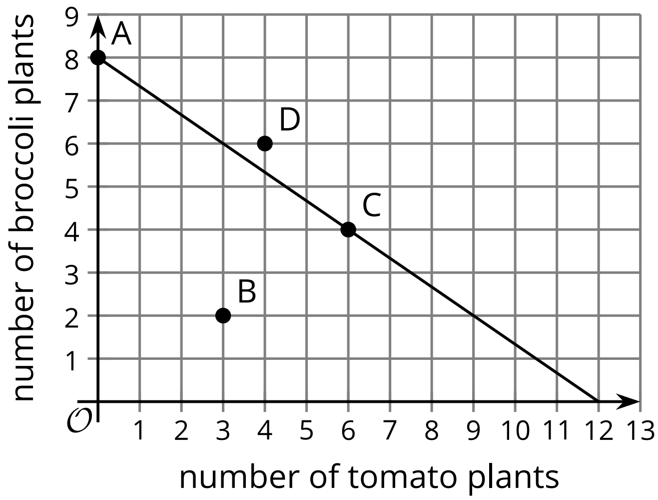
    b. Write an equation that would make it easy to find the rate at which the car was traveling in miles per hour (mph), r, if it traveled for t hr.

Travel Time (hr.)	Rate of Travel (mph)
5	
4.5	
3	
2.25	

6. A group of 280 elementary school students and 40 adults are going on a field trip. They are planning to use two different types of buses to get to the destination. The first type of bus holds 50 people and the second type of bus holds 56 people.

    Andre says that 3 of the first type of bus and 3 of the second type of bus will hold all of the students and adults going on the field trip. Is Andre correct? Explain your reasoning.

7. To grow properly, each tomato plant needs 1.5 square feet (sq. ft.) of soil and each broccoli plant needs 2.25 sq. ft. of soil. The graph shows the different combinations of broccoli and tomato plants in an 18 sq. ft. plot of soil.



Match each point to the statement that describes it.

Point
a. Point A
b. Point B
c. Point C
d. Point D

Statement
The soil is fully used when 6 tomato plants and 4 broccoli plants are planted.
Only broccoli was planted, but the plot is fully used and all plants can grow properly.
After 3 tomato plants and 2 broccoli plants were planted, there is still extra space in the plot.
With 4 tomato plants and 6 broccoli plants planted, the plot is overcrowded.

Unit 3, Lesson 3: Adding and Subtracting Complex Numbers

Warm-Up: Math Talk: Telescoping Sums

1. Find the value of these expressions mentally.

$2-2+20-20+200-200$

$100-50+10-10+50-100$

$3+2+1+0-1-2-3$

$1+2+4+8+16+32-16-8-4-2-1$

Guided Activity: The Complex Number System

A *complex number* is a number that can be written in the form $a+bi$, where a and b are real numbers. In the form $a+bi$, a is considered the **real** part of the complex number, and bi is the **imaginary** part of the complex number. Complex numbers where $a=0$ are often referred to as *pure imaginary numbers*.

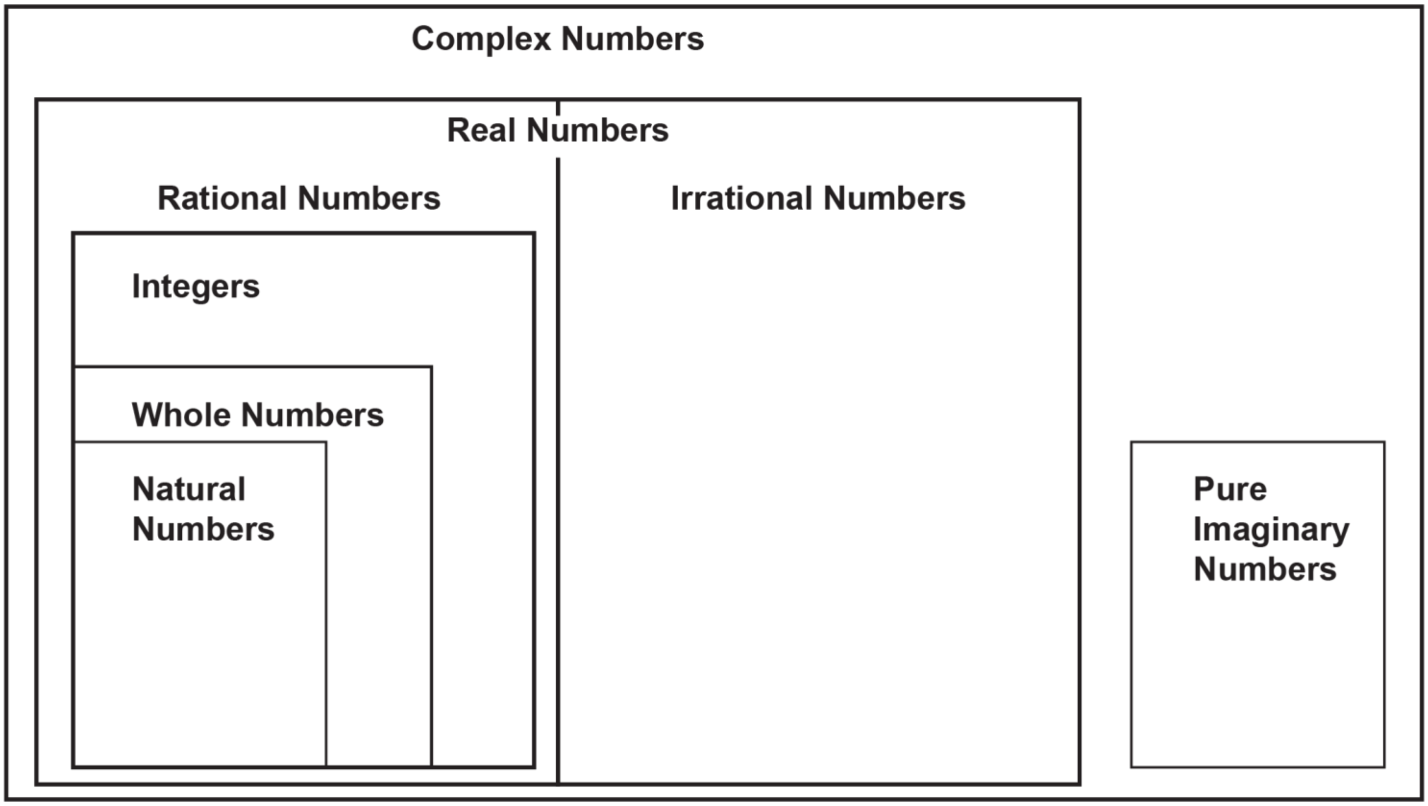
1. A list of numbers is shown.

$$3,\quad \frac{\sqrt[3]{3}}{2},\quad -2i,\quad 4+\sqrt{6}i,\quad -\frac{1}{6},\quad \pi-i,\quad \frac{1}{2}(1-\sqrt{-4}),\quad \sqrt[3]{-8},\quad 5i^2,\quad \frac{i}{3}$$

Sort the numbers into the categories shown in the table.

Complex Numbers $(a+bi)$		
Real Numbers $(b=0)$	**Pure Imaginary Numbers** $(a=0,\ b\neq 0)$	**Other Complex Numbers** $(a\neq 0,\ b\neq 0)$

The number system includes categories, or subsets, of numbers. With the introduction of additional number types, the system grows to represent the unique characteristics of each number. A Venn diagram representing the relationship between different number types within the complex number system is shown.



2. Classify each real number expression by writing them in the appropriate sections of the Venn diagram.

$$\sqrt{49},\ \frac{2}{3},\ -2\pi,\ -\frac{8}{4},\ 0,\ \frac{\sqrt{70}}{2},\ 4-\sqrt{4},\ 1.\overline{03}$$

3. Two new subsets of numbers have been introduced in this unit. Discuss with your partner whether an imaginary number is also a complex number.

Exploration Activity: Adding and Subtracting Complex Numbers

1. For each complex number, place a box around the real part, and circle the imaginary part.

$5 + 2i$ $2 - 3i$ $-5i + 23$ $10 + 2i$

2. Louis was asked to add and subtract 2 complex numbers, $2 + 5i$ and $3 - 8i$. His work is shown.

Add	$(2 + 5i) + (3 - 8i) = 5 - 3i$
Subtract	$(2 + 5i) - (3 - 8i) = 2 + 5i - 3 + 8i = -1 + 13i$

With your partner, explain what Louis did to add and subtract the 2 expressions.

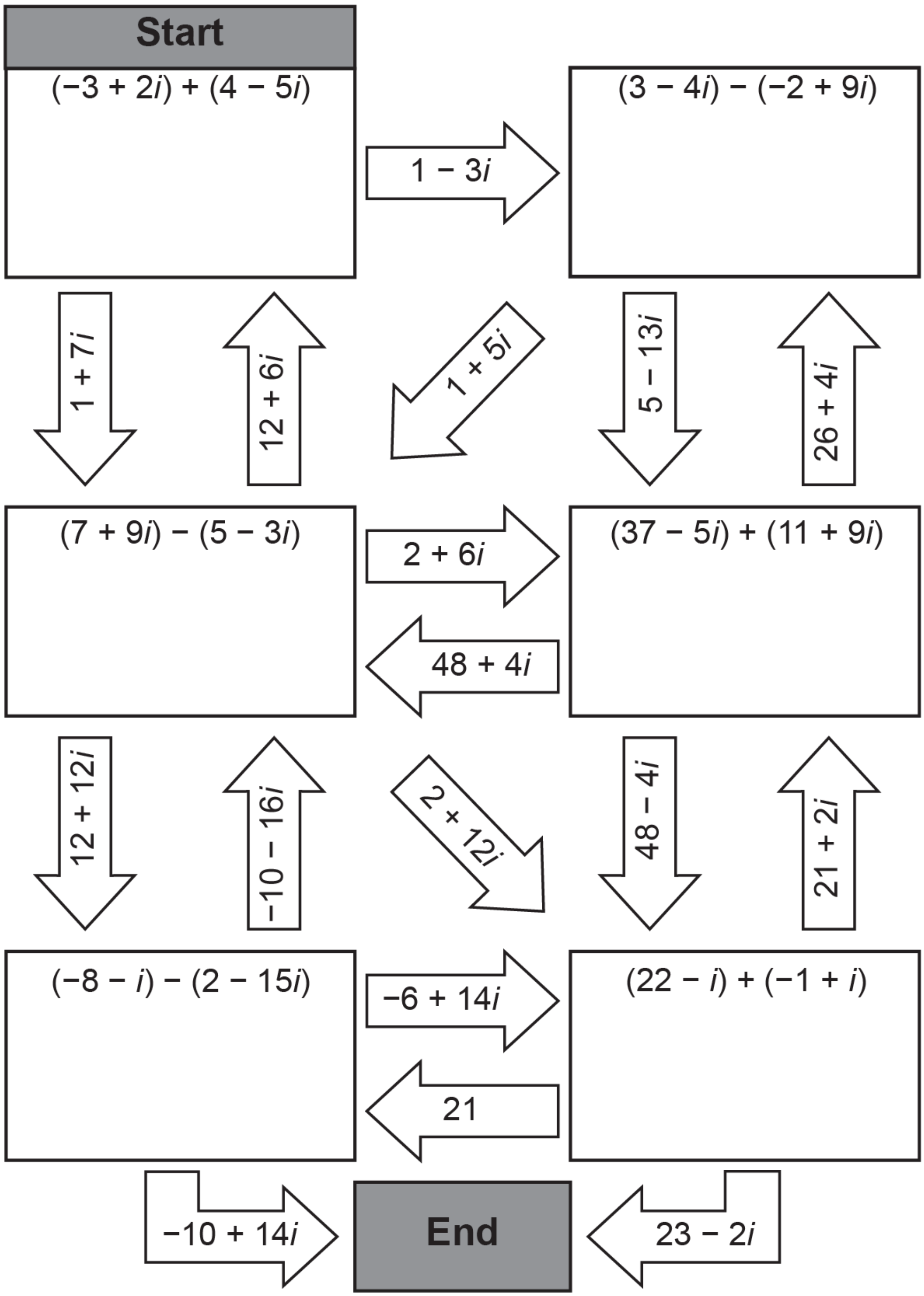
3. Perform the given operation for the complex numbers.

   a. $(3 + 5i) + (7 - 2i)$

   b. $(3 - 8i) - (6 + 12i)$

Collaborative Activity: Adding and Subtracting Complex Numbers Maze

1. Add or subtract each complex expression. Shade in the correct path to complete the maze from start to end.



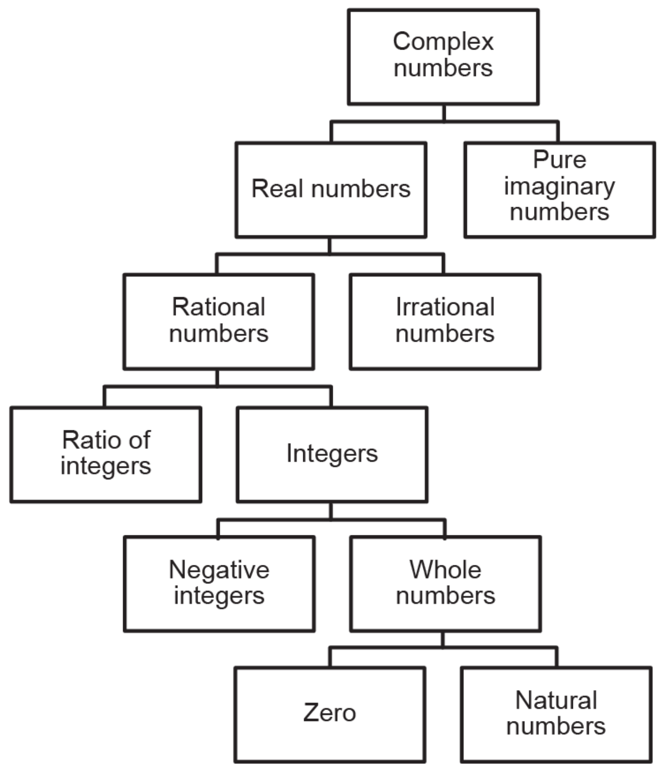
Lesson Summary

When a real number and an imaginary number are combined, the result is a *complex number*.

> A **complex number** is a number in the complex plane. It can be written as $a + bi$, where a and b are real numbers and $i^2 = -1$.

A complex number has 2 parts: the real part, a, and the imaginary part, bi.

The number system can be organized using a Venn diagram as shown in the lesson or as a hierarchy where each subset of numbers is shown below the broader number type. The complex number system's hierarchy is shown in the image.



Notice that real numbers and pure imaginary numbers are both subsets of complex numbers.

- Real numbers are complex numbers where $b = 0$.
- Pure imaginary numbers are complex numbers where $a = 0$.

The process of adding and subtracting complex numbers is similar to that of adding and subtracting polynomials. With polynomials, like terms are combined. With complex numbers, like parts are combined.

The sum and difference of the complex numbers $2 + 3i$ and $4 + 5i$ are shown.

$$(2 + 3i) + (4 + 5i) = (2 + 4) + (3i + 5i) = 6 + 8i$$

$$(2 + 3i) - (4 + 5i) = (2 - 4) + (3i - 5i) = -2 - 2i$$

In general, the sum (or difference) of complex numbers results in the sum (or difference) of the coefficients of the real parts and the coefficients of the imaginary parts of the complex numbers.

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

Practice Problems

1. Which expression is equivalent to $(3+9i)-(5-3i)$?

   A. $-2-12i$

   B. $-2+12i$

   C. $15+27i$

   D. $15-27i$

2. What are a and b when you write $\sqrt{-16}$ in the form $a+bi$, where a and b are real numbers?

   A. $a=0,\ b=-4$

   B. $a=0,\ b=4$

   C. $a=-4,\ b=0$

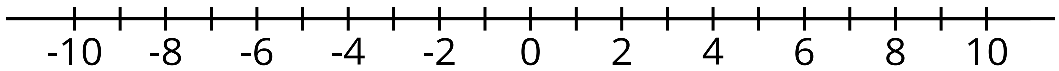
   D. $a=4,\ b=0$

3. Fill in the boxes to make a true statement:

   $(\square-3i)-(15+\square i)=7-12i$

Review Problems

4. Plot each number on the real number line, or explain why the number is not on the real number line.



a. $\sqrt{16}$

b. $-\sqrt{16}$

c. $\sqrt{-16}$

d. $56^{\frac{1}{2}}$

e. $-56^{\frac{1}{2}}$

f. $(-56)^{\frac{1}{2}}$

5. Which expression is equivalent to $\sqrt{-4}$?

A. $-2i$

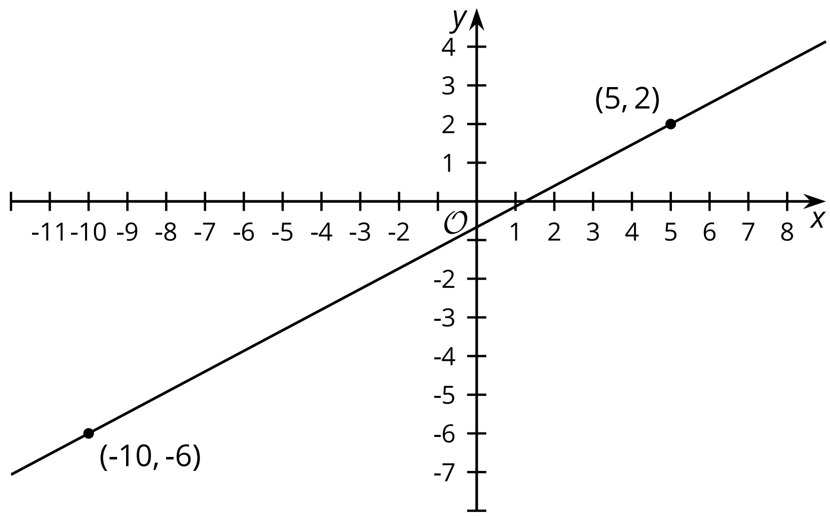
B. $-4i$

C. $2i$

D. $4i$

Unit 8, Lesson 1: Revisiting Equations of Lines

Warm-Up: Remembering Slope



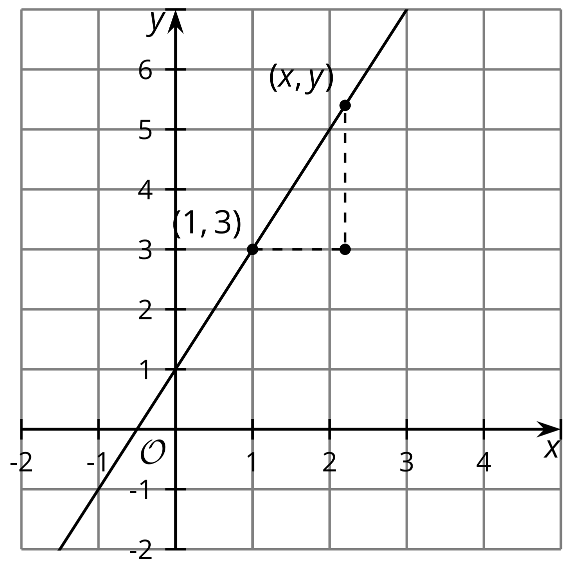
The slope of the line in the image is $\frac{8}{15}$. Explain how you know this is true.

Exploration Activity: Building an Equation for a Line

Recall from 8th grade that equations representing linear relationships can be written in *slope-intercept form*.

- Slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.
- The slope, m, is the ratio of the change in the vertical direction to the change in the horizontal direction between 2 points, often expressed as $\frac{\Delta y}{\Delta x}$.

1. The graph of a linear equation is shown on the coordinate plane.



   a. Write an equation that shows the slope between the points $(1, 3)$ and (x, y) is 2.

   b. Consider the equation $y - 3 = 2(x - 1)$. How does it relate to the equation you wrote?

   c. Rewrite the equation from part A in slope-intercept form.

2. Another linear equation can be represented by the equation $\frac{y-7}{x-5} = \frac{1}{2}$.

   a. What point do you know is on this line?

   b. What is the slope of the line?

   c. Rewrite the equation in slope-intercept form.

Collaborative Activity: Equations of Lines

1. Josiah and Claudia each wrote the equation of a line that passes through the point $(-7, 5)$ with slope 3 in slope-intercept form. Their work is shown.

Josiah's Work	Claudia's Work
$\frac{y-5}{x-(-7)} = 3$ $\frac{y-5}{x+7} = 3$ $y - 5 = 3(x + 7)$ $y - 5 = 3x + 21$ $y = 3x + 26$	$y = mx + b$ $5 = (3)(-7) + b$ $5 = -21 + b$ $26 = b$ $y = 3x + 26$

a. Discuss with your partner the method Josiah used to write the equation of the line in slope-intercept form. Summarize his method.

b. Discuss with your partner the method Claudia used to write the equation of the line in slope-intercept form. Summarize her method.

c. Use each of their methods to write the equation of a line that passes through the point $(6, 1)$ with slope $\frac{1}{2}$ in slope-intercept form.

Josiah's Method	Claudia's Method

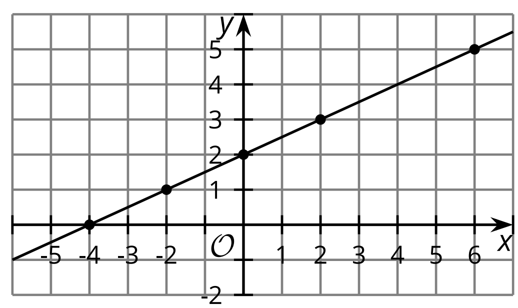
2. Write an equation of a line in slope-intercept form from each description.

a. The line passing through point $(-2, 8)$ with slope $\frac{4}{5}$

b. The line passing through point $(0, 7)$ with slope $-\frac{7}{3}$

c. The line passing through point $\left(\frac{1}{2}, 0\right)$ with slope -1

d. The line in the image



3. Determine the slope and a point that each line passes through using the structure of each equation.

a. $\frac{y-5}{x+4} = \frac{3}{2}$

b. $y = 5x - 2$

c. $y = -2\left(x - \frac{5}{8}\right)$

Lesson Summary

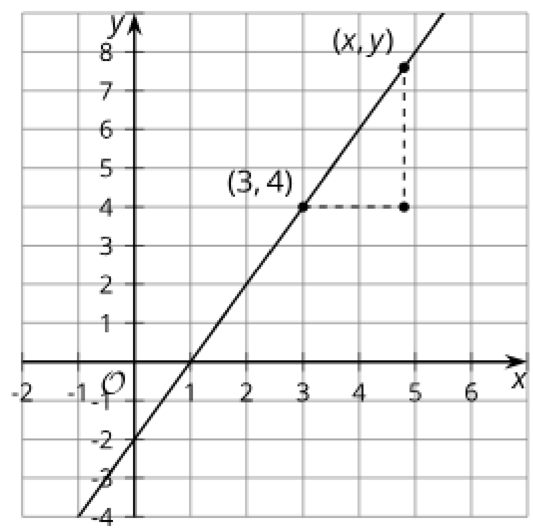
This lesson focuses on writing the equation of a line in slope intercept form, $y = mx + b$, where m is the *slope* of the line and b is the y-*intercept* of the line.

The **slope** is the ratio of the change in the vertical direction (y- direction) to the change in the horizontal direction (x-direction), often expressed as $\frac{\Delta y}{\Delta x}$.

The $\boldsymbol{y}$**-intercept** is the value of y at the point where a line or graph intersects the y-axis. The value of x is 0 at this point.

There are an infinite number of points, (x, y), that satisfy the equation of a line.

The line shown on the coordinate plane can be defined as the set of points that includes the point $(3, 4)$ and has a slope of 2. The given information can be used to write an equation of the line.



- One method is using slope, $m = \frac{\Delta y}{\Delta x}$, read, "Slope is the change in y over the change in x." The points $(3, 4)$ and (x, y) can be used along with the slope to write the equation shown. This equation can then be rearranged into slope-intercept form.

$$\frac{y-4}{x-3} = 2$$

$$y - 4 = 2(x - 3)$$

$$y - 4 = 2x - 6$$

$$y = 2x - 2$$

- Another method that can be used to write an equation of a line that passes through point $(3, 4)$ with a slope of 2 is to substitute the (x, y) values of the known point and the slope into slope-intercept form to find the value of b. Then, use the values of m and b to write the equation.

$$y = mx + b$$

$$4 = 2(3) + b$$

$$4 = 6 + b$$

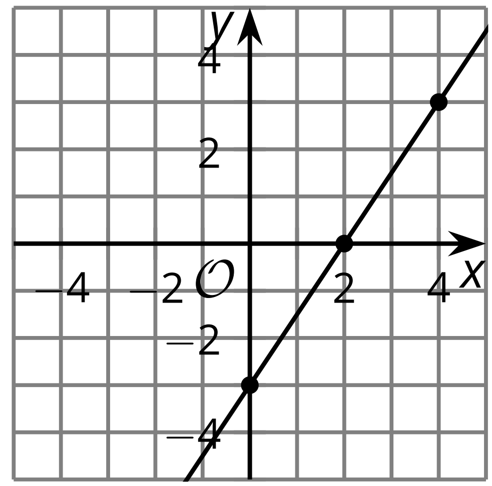
$$-2 = b$$

$$y = 2x - 2$$

Notice that both methods result in the same equation in slope-intercept form.

Practice Problems

1. Select all of the equations that represent the graph shown.



- ☐ $3x - 2y = 6$
- ☐ $y = \frac{3}{2}x + 3$
- ☐ $y = \frac{3}{2}x - 3$
- ☐ $\frac{y-3}{x-4} = \frac{3}{2}$
- ☐ $\frac{y-6}{x-2} = \frac{3}{2}$

2. A line with slope $\frac{3}{2}$ passes through the point $(1, 3)$.

   a. Explain why $(3, 6)$ is on this line.

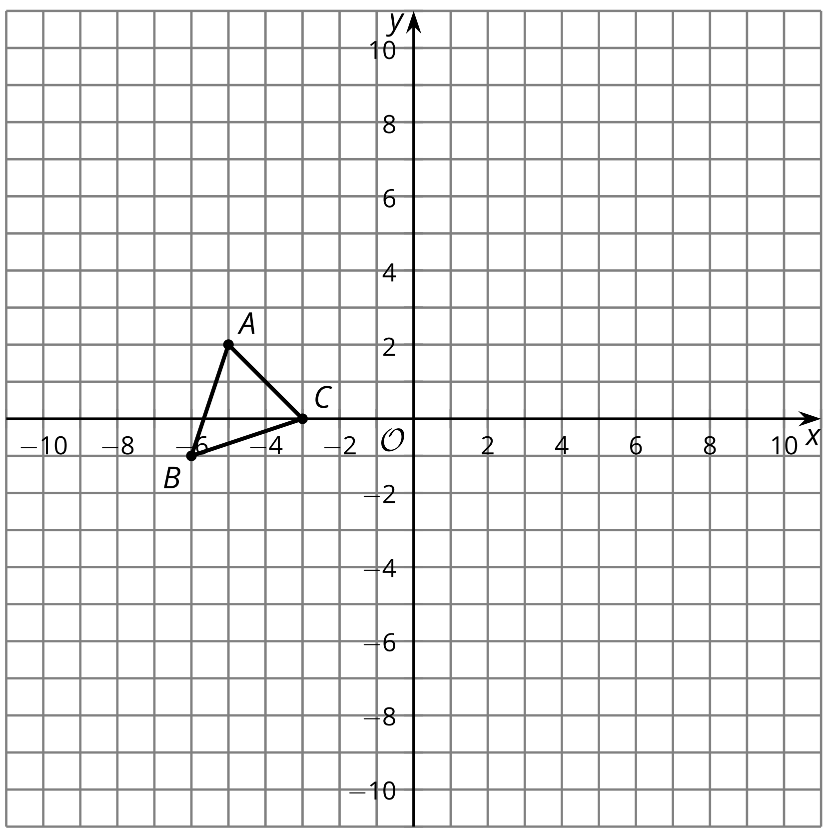
   b. Explain why $(0, 0)$ is not on this line.

   c. Is the point $(13, 22)$ on this line? Explain why or why not.

3. Write an equation of the line that passes through $(1, 3)$ and has a slope of $\frac{5}{4}$.

Review Problem

4. Reflect triangle ABC over the line $x = -6$. Translate the image by the directed line segment from $(0, 0)$ to $(5, -1)$.



What are the coordinates of the vertices in the final image?