Unit 10, Lesson 4: Determining the Probability of the Complement of an Event

Warm-Up: What Song?

There are 50 songs in a playlist set to play on shuffle, so the order of the songs played will be random. Genre-wise, 20 of the songs are pop, 15 songs are hip-hop, 10 songs are rock, and 5 are country.

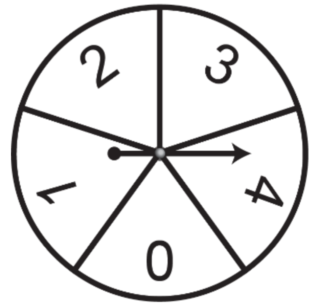
1. What is the probability that the first song played will be the first song in the playlist?

2. What is the probability that the genre of the first song played will be hip-hop?

3. What is the probability that the first song played won't be a pop song?

Exploration Activity: Other Outcomes

A game spinner with 5 equal sections is shown.



1. List all of the possible outcomes.

2. Event A is spinning an even number.

    a. List all outcomes in event A.

    b. $P(\text{A}) =$ ____________

The *complement* of event A is all of the other outcomes not in event A. The complement of event A is notated as “not A.”

    c. List all outcomes not in event A.

    d. $P(\text{not A}) =$ ____________

A bag contains 10 colored marbles. The color distribution is shown in the table.

Green	**Blue**	**Purple**	**Pink**
4	3	2	1

3. Event M is randomly drawing a pink or green marble from the bag.

    a. $P(\text{M}) =$ ____________

    b. List all outcomes not in event M.

c. $P(\text{not M}) =$ ____________

4. What is the sum of $P(\text{A})$ and $P(\text{not A})$?

5. What is the sum of $P(\text{M})$ and $P(\text{not M})$?

6. Using your answers from questions 5 and 6, what can you conclude about the probabilities of an event and its complement?

Collaborative Activity: Finding Complements of Events

1. Event A is randomly choosing a letter A from the word *QUADRANT*.

   a. Find $P(\text{A})$.

   b. Find $P(\text{not A})$. Show or explain your thinking.

   c. What is another way to determine $P(\text{not A})$ in addition to the method used to answer the previous question?

2. The table shows the distribution of chocolate candies in a candy jar.

Milk	Dark	White
7	3	15

a. Find $P(\text{milk})$.

b. Find $P(\text{not milk})$.

3. If $P(\text{C}) = 25\%$, determine $P(\text{not C})$.

4. If $P(\text{not D}) = \frac{14}{25}$, determine $P(\text{D})$.

5. If $1 - P(\text{B}) = 0.99$, determine $P(\text{B})$.

6. When asked to find the complement of $P(\text{E}) = 0.06$ as a percentage, Ronan incorrectly stated $P(\text{not E}) = 40\%$. Explain the error in Ronan's thinking.

Lesson Summary

Recall that an *event* is the desired outcome in a random experiment. The *complement* of an event is all of the other (undesired) outcomes that can occur. In some random experiments, like spinning a spinner with 5 sections or rolling a number cube, there may be multiple possible outcomes, but when exploring complements, the events are defined as the desired and undesired outcomes only.

The sum of the probabilities of an event and its complement is always 1. Therefore, the complement of an event, A, can be found using the equation $1 - P(\text{A}) = P(\text{not A})$. It is also true that $1 - P(\text{not A}) = P(\text{A})$ and $P(\text{A}) + P(\text{not A}) = 1$.

Practice Problems

1. A playlist contains 25 songs from a variety of artists. The artist distribution is shown in the table.

Pink Floyd	Bob Marley	Elvis	The Beatles
3	9	5	8

Event A is randomly playing a Beatles song first. Find $P(\text{not A})$.

2. Event B is randomly choosing a letter E from the word *PRETZELS*.

   a. List all outcomes of event B.

   b. Find $P(\text{B})$.

   c. Find $P(\text{not B})$. Show or explain your thinking.

3. The table shows the distribution of cookie flavors in a cookie jar.

Chocolate Chip	Oatmeal	Sprinkle
13	17	20

a. Find $P(\text{sprinkle})$.

b. Find $P(\text{not oatmeal})$.

Review Problems

4. Select **all** the expressions that are equivalent to $4b$.

- ☐ $b+b+b+b$
- ☐ $b+4$
- ☐ $2b+2b$
- ☐ $b \cdot b \cdot b \cdot b$
- ☐ $b \div \frac{1}{4}$

5. Elena is designing a logo in the shape of a parallelogram. She wants the logo to have an area of 12 square inches (sq. in). She draws bases of different lengths and tries to compute the height for each.

a. Write an equation Elena can use to find the height, h, for each value of the base, b.

b. Use your equation to find the height of a parallelogram with base 1.5 inches (in.).

Unit 7, Lesson 3: Factoring Linear Expressions

Warm-Up: Number Talk: Parentheses

Find the value of each expression mentally.

1. $2 + 3 \cdot 4$

2. $(2 + 3)(4)$

3. $2 - 3 \cdot 4$

4. $2 - (3 + 4)$

Collaborative Activity: Exploring Equivalent Expressions Using Common Factors

Review with your partner what you learned about common factors in prior grades. Then, work together to complete the problems that follow.

In multiplication expressions, the numbers being multiplied are called *factors*, and the result is called the *product*.

Numbers can have *common factors*. A common factor of 2 numbers is a number that divides evenly into both numbers.

- For example, $1, 3, 5, 9, 15$, and 45 are factors of 45, and $1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30$, and 60 are factors of 60.
- The common factors of 45 and 60 are $1, 3, 5$, and 15.

The *greatest common factor (GCF)* is the largest factor that the values share.

- For example, 15 is the GCF of 45 and 60.

Common factors can be used to rewrite numerical expressions.

1. Consider the expression $24 + 32$.

    a. List all the common factors of 24 and 32.

    b. Complete the steps to rewrite the sum of 24 and 32 using their GCF.

    $24 + 32$

    $(8 \times \_\_\_\_) + (8 \times \_\_\_\_)$

    $8(\_\_\_\_ + \_\_\_\_)$

    c. Choose another **common factor** of 24 and 32 to write a different expression equivalent to $24 + 32$.

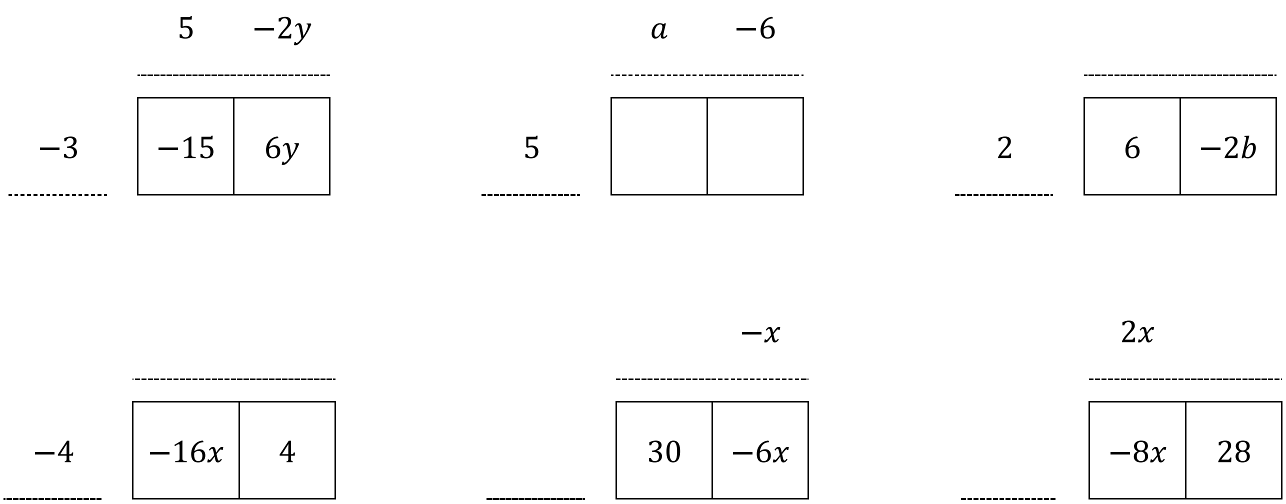
    $24 + 32$

    $(\_\_\_\_ \times \_\_\_\_) + (\_\_\_\_ \times \_\_\_\_)$

    $\_\_\_\_(\_\_\_\_ + \_\_\_\_)$

Common factors can also be used to rewrite equivalent algebraic expressions.

2. Use your understanding of common factors to complete the area models shown. In each area model, the factors are along the dotted lines, and their products are in the rectangular area. The first model has been completed as an example.



An area model can be used to show the relationship between equivalent forms of algebraic expressions.

In the area models in the previous problem, the factored form of an expression is the (width) × (length) of the larger rectangle. An equivalent expression in expanded form can be represented by the sum of the areas of the smaller rectangles.

3. Complete the table using the area models from the previous problem.

4. Compare your work with your partner's. Discuss any differences, and make any corrections, if needed.

Factored Form	Expanded Form
$-3(5-2y)$	$-15+6y$
$5(a-6)$	
	$6-2b$
	$-16x+4$
	$30-6x$
	$-8x+28$

Guided Activity: Applying Properties of Operations to Factor and Expand Linear Expressions

Using an area model to rewrite a linear expression in an equivalent factored form is a method of **factoring an expression**. Factoring is another way of applying the distributive property of multiplication over addition.

1. Without using area models, complete the table so that each row includes equivalent linear expressions.

Factored Form	Expanded Form
$-4(2w-5)$	
$-(2-3y)$	
	$20x-10$
$4(a-11)$	
	$10-25b$
$-2(3-z)$	

2. Consider the expression $24x-8$.

   For the expression $24x-8$, Pao and Camiya wrote different factored forms of the expression, as shown.

Pao's Expression	Camiya's Expression
$4(6x-2)$	$8(3x-1)$

   a. Explain who wrote a correct equivalent expression in factored form.

b. What is another equivalent expression using a different common factor?

c. Which expression is equivalent to $24x - 8$, using the GCF of $24x$ and -8?

Properties of operations can be used in different ways to create equivalent expressions.

- The distributive property can be used to expand an expression. For example, the expression $3(x + 5)$ can be rewritten as $3(x) + 3(5)$ or $3x + 15$.
- The distributive property can also be used to factor an expression. For example, the expression $8x + 12$ can be rewritten as $2(4x + 6)$ or $4(2x + 3)$.

Equivalent expressions can also be generated using the commutative property of addition or multiplication as well as the distributive property. An example is shown.

$$(4x + 6)2 \qquad\qquad 4(3 + 2x)$$

3. Complete the following to factor the expression $12d - 36$.

a. Choose a common factor of the terms $12d$ and -36. Circle your choice.

$$2 \qquad 3 \qquad 4 \qquad 6 \qquad 12$$

b. What is the result of dividing $12d$ by the common factor selected?

c. What is the result of dividing -36 by the common factor selected?

d. Use your answers to parts A–C to rewrite an expression equivalent to $12d - 36$ in factored form.

$$12d - 36 = \_\_\_\_(\_\_\_\_ + \_\_\_\_)$$

e. Choose a different common factor from part A, and use it to write a different equivalent expression.

$$12d - 36 = \_\_\_\_(\_\_\_\_ + \_\_\_\_)$$

4. Several expressions are shown.

$8(3+m)$	$2(8m+12)$	$4(2m+6)$
$(2m+8)3$	$(1+3)8m$	$(12+4m)2$

a. Circle the expressions equivalent to $8m+24$.

b. Put an asterisk (*) next to the equivalent expression that has the GCF of $8m$ and 24 as 1 of its factors.

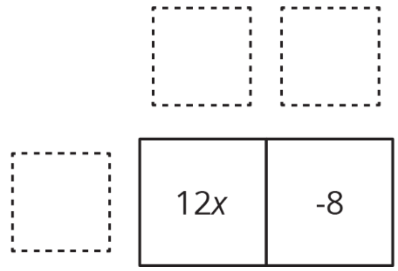
c. Put a smiley face (☺) next to an expression where more than 1 property of operations can be used to show it is equivalent to $8m+24$.

Lesson Summary

Properties of operations can be used in different ways to generate equivalent expressions. The distributive property can be used to expand an expression. For example, $3(x+5)=3x+15$. The distributive property can also be used in the other direction to factor an expression. For example, $8x+12=4(2x+3)$.

The work of using the distributive property to factor the expression $12x-8$ can be represented by an area model.

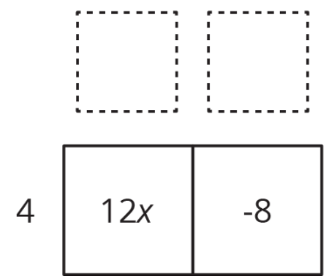
- The terms of the product go inside the area of the rectangle, as shown.



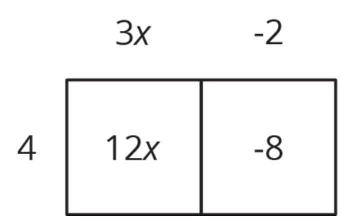
- Use the expression to find the *greatest common factor* of both terms.

> The **greatest common factor** (GCF) of 2 or more whole numbers is the largest whole number that evenly divides the given whole numbers.

The terms $12x$ and -8 each have a GCF of 4. The GCF is placed on one side of the large rectangle to represent one dimension of the rectangle, as shown.



- To determine the other dimension of the rectangle using the area model, think "4 times *what* is $12x$?" and "4 times *what* is -8?" and write the other factors along the top dimension of the rectangle, as shown.



The factors resulting in a product of $12x-8$ are 4 and $(3x-2)$. So, $12x-8$ is equivalent to $4(3x-2)$.

Practice Problems

1. Fill in the blanks of the equation to make it true.

$75a+25b=$ _____ (_____$a+b$)

2. Rewrite each expression in an equivalent factored form.

   a. $-15z + 20 =$ ______________________

   b. $4x - 32 =$ ______________________

   c. $-27 - 12d =$ ______________________

   d. $35 + 28g =$ ______________________

3.

   a. Expand to write an equivalent expression: $\frac{-1}{4}(-8x + 12y)$

   b. Factor to write an equivalent expression: $36a - 16$

Review Problem

4. Elena makes her favorite shade of purple paint by mixing 3 cups (c.) of blue paint, $1\frac{1}{2}$ c. of red paint, and $\frac{1}{2}$ of a c. of white paint. Elena has $\frac{2}{3}$ of a c. of white paint.

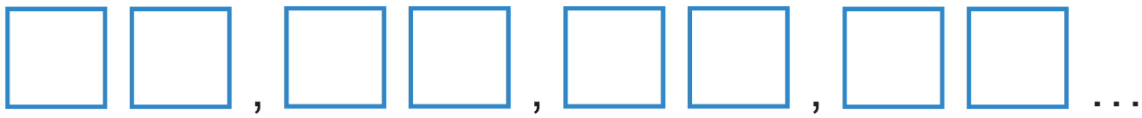
   a. Assuming she has enough red paint and blue paint, how much purple paint can Elena make?

   b. How much blue paint and red paint will Elena need to use with the $\frac{2}{3}$ of a c. of white paint?

Unit 5, Lesson 3: Domain and Range

Warm-Up: Create a Pattern

1. Place a digit in each blue box to create a pattern where the next number changes by the same amount each time. Use only the digits 0 to 9, without repeating any digits.



Guided Activity: Introducing Domain and Range

1. Consider the statements shown.

   $12 \div 3 = 4$ because $12 = 4 \cdot 3$.

   $4 \div x = 1$ because $4 = x \cdot 1$.

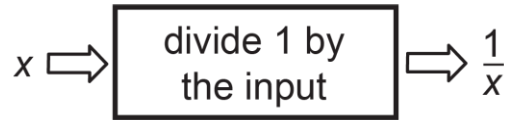
   $6 \div 0 = y$ because $6 = y \cdot 0$.

   a. In the second statement, what value can be used in place of x so that the statement is true?

   b. In the third statement, what value can be used in place of y so that the statement is true?

The inputs of a relation are described as its *domain*. The domain of a relation can be written as a list of values or as an inequality. In some cases, 1 or more values are not allowed in the domain of a relation. The domain for a relation can be represented using inequalities that include all possible input values defined as x.

2. Consider the rule shown for a relation.



a. Complete the table of values using the relation.

x	$-\frac{3}{7}$	1	$\frac{4}{5}$	0	12
y	$-\frac{7}{3}$				

b. Identify any values for x that are not allowed in the domain of the relation.

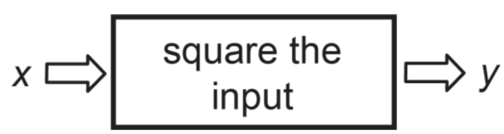
c. Work with your partner to determine if there is another input that is not allowed for the relation $\frac{1}{x}$.

d. Complete the statements to identify the domain of the relation.

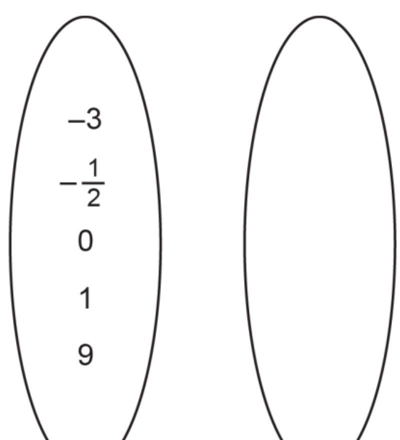
The domain of the relation is $x >$ _____ or $x <$ _____. These 2 inequalities can also be written as one statement, $x \neq$______.

The outputs of a relation are described as its *range*. The range of a relation can be written as a list of values or as an inequality. In some cases, 1 or more values are not possible in the range of a relation. The range of a relation can be represented using inequalities that include all possible output values defined as y.

3. Consider the rule shown for a relation.



a. Complete the mapping diagram for the relation.



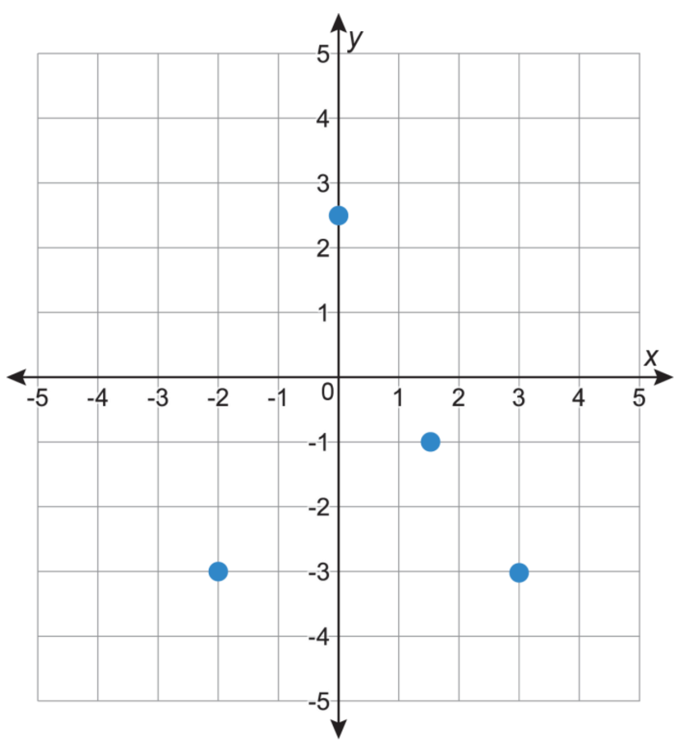
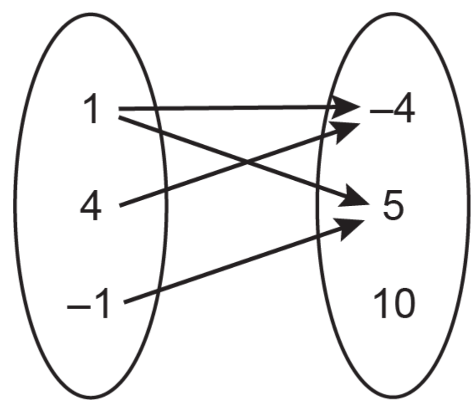
b. Based on the rule, what type of output values are not possible for this relation?

c. Write an inequality to represent the range of this relation.

y ◯ ____

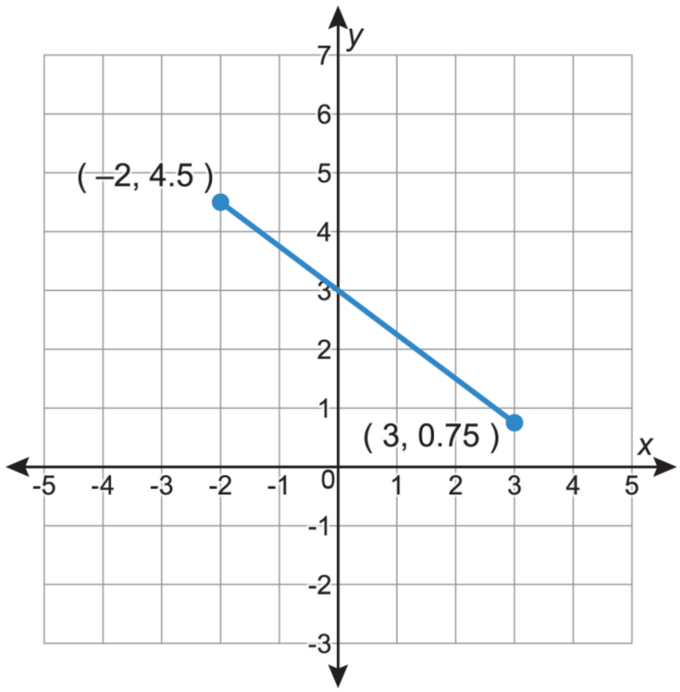
In some cases, the domain and range cannot be written as an inequality because the relation includes only specific values. In such cases, the domain and range are written as a set of numbers within brackets, { and }.

4. For each relation, write the domain and range as a set of numbers.

Relation A	**Relation B**
	
Domain: ____________	Domain: ____________
Range: ____________	Range: ____________

There are also instances where the domain and range of a relation can be represented by a *compound inequality*. For example, $-4 \leq x \leq \frac{3}{5}$ is a compound inequality that reads as "-4 is less than or equal to x, which is less than or equal to $\frac{3}{5}$." This inequality is true for any value of x between -4 and $\frac{3}{5}$, including either of those values.

5. A graph is shown.



a. Represent the domain and range of the graph using compound inequalities.

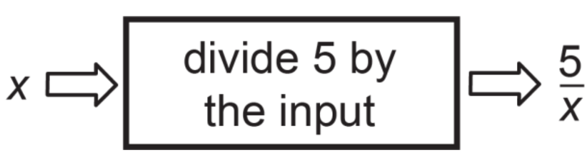
Domain: ________________________

Range: _________________________

b. Compare this graph to relation A in the previous problem. Discuss with your partner why the domain and range of this graph could not be represented by writing the elements of each in a list.

Collaborative Activity: Identifying Domain and Range of a Relation

1. Identify the domain and range of each relation. Represent the domain and range as lists or using inequalities.

<table>
<tr><th>Relation</th><th>Domain</th></tr>
<tr><td rowspan="3"><table><tr><th>x</th><th>y</th></tr><tr><td>-4.7</td><td>7</td></tr><tr><td>2.4</td><td>4.7</td></tr><tr><td>-1.6</td><td>3.9</td></tr><tr><td>8</td><td>10</td></tr></table></td><td></td></tr>
<tr><th>Range</th></tr>
<tr><td></td></tr>
<tr><th>Relation</th><th>Domain</th></tr>
<tr><td rowspan="3">$\{(2.3, 7.4), (3.8, 6), (1.9, 5.2), (2.3, 8)\}$</td><td></td></tr>
<tr><th>Range</th></tr>
<tr><td></td></tr>
<tr><th>Relation</th><th>Domain</th></tr>
<tr><td rowspan="3"></td><td></td></tr>
<tr><th>Range</th></tr>
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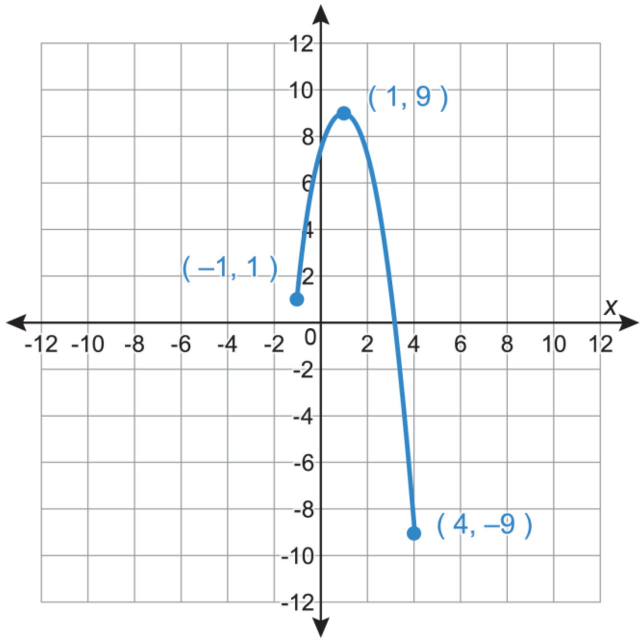
</table>

Relation	Domain
	
	Range

Lesson Summary

The *domain* of a relation can be represented as a list or an inequality of all possible x-values that can be used in the relation. The *range* of a relation can be represented as a list or an inequality of all possible y-values that can be used in the relation.

Domain is the complete set of possible values of the input of a function or relation. The domain may vary depending on the context.

Range is the complete set of possible values of the output of a relation or function.

When organized in a list, the values are written in increasing order without duplicates. When represented as an inequality, a *compound inequality* is often used.

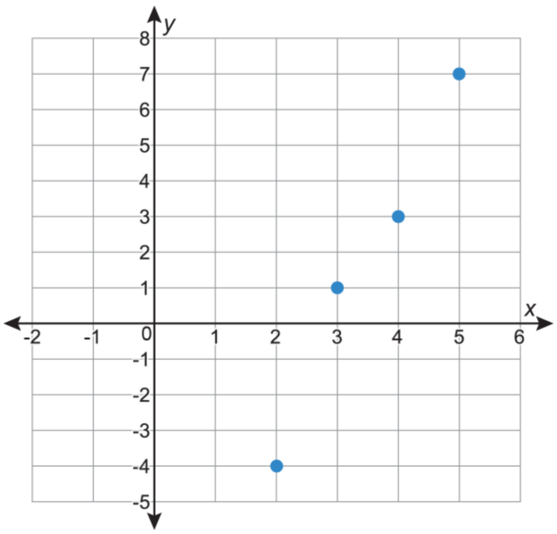
A **compound inequality** is a conjunction of two or more inequalities.

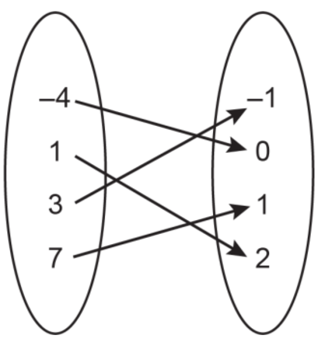
As you continue learning the different representations of input-output relations, you will be able to apply the concepts of domain and range in comparing different features of relationships between the variables.

Practice Problems

1. Five relations are shown. Select **all** of the relations with the domain $\{-4, 1, 3, 7\}$.

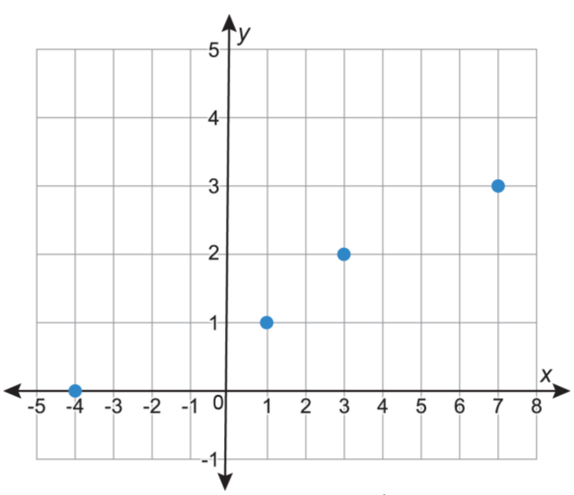
☐ $\{(7,2), (1,4), (3,4), (-4,-1)\}$

☐ 

☐ 

☐

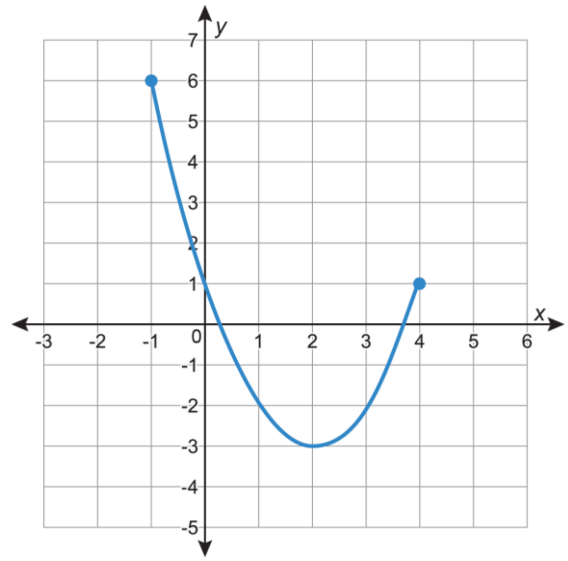
x	-4	0	2	6
y	-4	3	1	7

☐ 

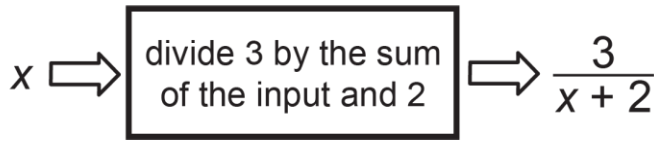
2. The graph of a relation is shown.

   a. Identify the domain of the relation.

   b. Identify the range of the relation.



3. A rule is shown for a relation.



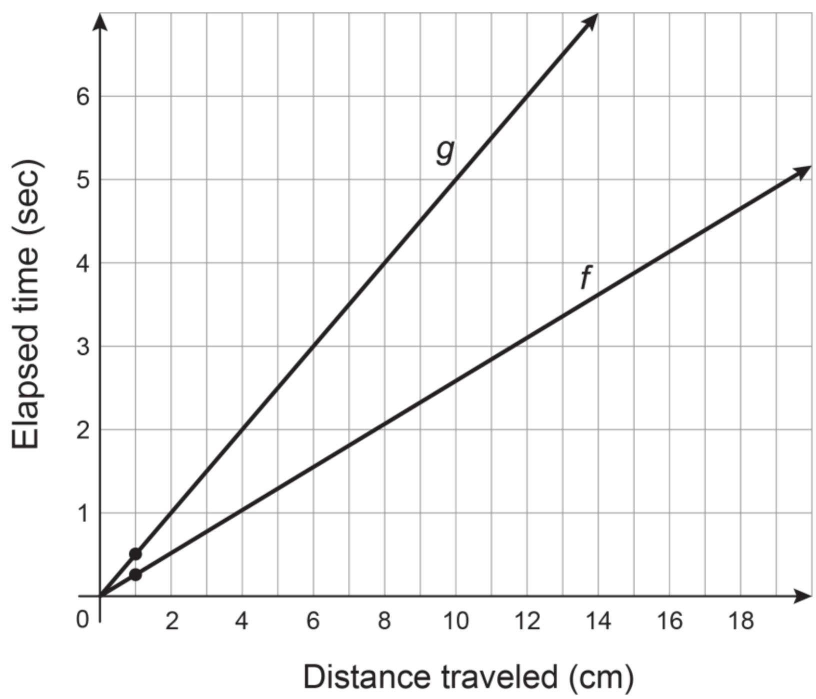
   Identify the domain of the relation.

Review Problems

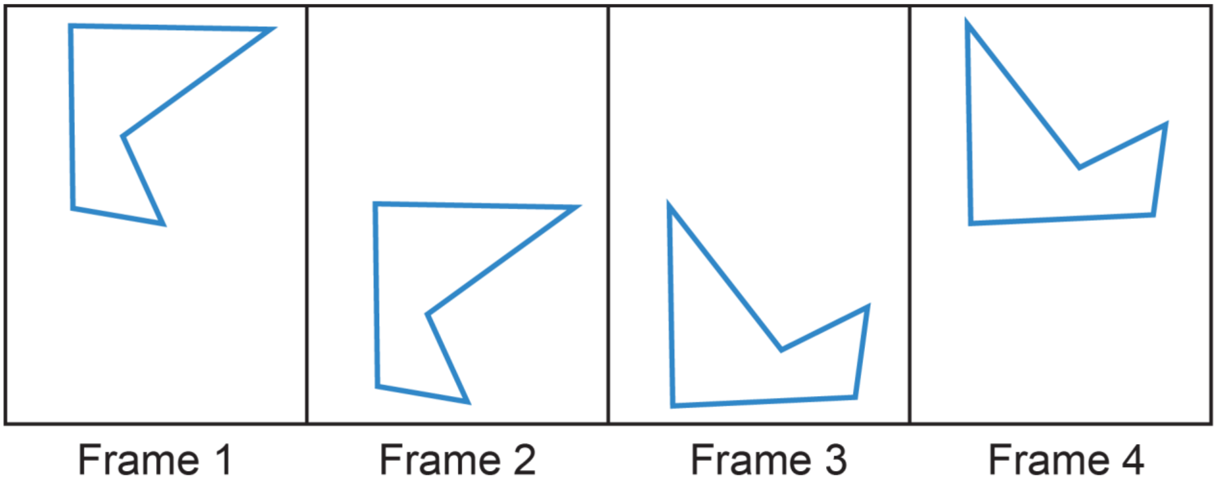
4. The graphs represent the positions of 2 turtles in a race.

   a. On the same axes, draw a line for a third turtle that is going half as fast as the turtle represented by line g.

   b. Explain how your line shows that the turtle is going half as fast.



5. Four successive positions of a shape are shown.



Describe how the shape moves from . . .

a. . . . frame 1 to frame 2.

b. . . . frame 2 to frame 3.

c. . . . frame 3 to frame 4.